Mixed states in Rabi waves and quantum nanoantennas

G. Ya. Slepyan,[1] Y. D. Yerchak,[1,*] S. A. Maksimenko,[1] A. Hoffmann,[2] and F. G. Bass[3]
[1]*Institute for Nuclear Problems, Belarus State University, Bobruiskaya 11, 220050 Minsk, Belarus*
[2]*Institut für Festkörperphysik, Technische Universität Berlin, Hardenbergstrasse 36, D-10623 Berlin, Germany*
[3]*Department of Physics, Bar-Ilan University, IL-52900 Ramat-Gan, Israel*

(Received 16 February 2012; revised manuscript received 27 April 2012; published 28 June 2012)

The mixed states of Rabi waves in a one-dimensional chain of two-level quantum dots (QDs) with tunnel coupling between neighbor QDs are theoretically considered. The propagation of Rabi waves is described by a 2×2 statistical operator depending on two spatial and one temporal variables. For a statistical operator the generalized Bloch equations are derived. The eigenmodes of the statistical operator are considered, the general solution of the initial problem is obtained, and the frequency spectrum of the induced current is investigated. The high-frequency part of the spectrum corresponds to a Mollow triplet, but is distinct from the last one as it is the continuous spectral band located on both sides of the transition frequency. The low-frequency part is caused by tunnel current and includes a dc-component and continuous ac-band in the vicinity of the Rabi oscillation frequency. The application of Rabi waves to the electrically controlled quantum nanoantennas of terahertz range is considered.

DOI: 10.1103/PhysRevB.85.245134 PACS number(s): 78.67.Hc, 71.10.Li, 73.40.Gk, 73.21.La

I. INTRODUCTION

Rabi oscillations are oscillating transitions of a two-level quantum system between its stationary states under the effect of an oscillating driving field. Rabi oscillations were first predicted in Ref. 1 as the periodical change of a nuclear spin's orientation in the radiofrequency magnetic field. Later the effect was found not only in spin systems, but in another types of oscillators, such as electromagnetically driven atoms.[2] Note that the effect of oscillating transitions takes place only in the strong coupling regime of a quantum oscillator with external field, which thus can not be considered as a small perturbation.

There are different models of Rabi oscillations. One of them is the quasiclassical model which implies that the atom interacts with classical light, which is not influenced by quantum transitions in the atom. Temporal dynamics of the inversion reduces to harmonic oscillations with the Rabi frequency.[3] The Jaynes-Cummings model accounts for the quantum-light state changes under the effect of the oscillator, and leads to the concept of the radiation-dressed atom.[3] The temporal dynamics of the inversion in the case of the coherent state leads to the well-known picture of collapses and revivals.

In spatially delocalized systems, such as quantum wires, atom chains, and so on, collective effects come into play. Some of them, such as the correlated spontaneous emission of a single photon,[4,5] quantum interference in cooperative Dicke emission,[6] the collective Lamb shift in single-photon superradiance,[7] finite time disentanglement via spontaneous emission,[8] directed spontaneous emission,[9] and so on, occur even in the weak coupling regime. In the strong coupling regime, collective effects manifest themselves as Rabi oscillations of quasiparticles moving in space. These quasiparticles are characterized by nonzero quasimomentum and the energy dependence on that quasimomentum. The Hopfield polariton in the strong coupling regime[10–13] can serve as an example. Such quasiparticles can form wavepackets and Rabi oscillations in these wavepackets suffer from diffraction broadening.

Some other possibilities appear in one-dimensional (1D) structures [for example, atomic or quantum dot (QD) chains], interacting with oblique incident light. If tunneling between neighboring QDs takes place then, distinct from Hopfield polaritons, indirect transitions corresponding to both energy and quasimomentum exchanges between light and the charge carrier are possible in the system.[14,15] This leads to the spatial propagation of Rabi oscillations in the form of plane waves and wavepackets (i.e., Rabi waves).[14,15] The Rabi waves take place both for classical[14] and quantum light.[15] In the later case, the collapses and revivals for the coherent state of light propagate in space. As was shown in Refs. 14 and 15, together with energy and quasimomentum, the Rabi waves transfer single exciton-exciton and exciton-photon correlations, which is of interest for information transmission and processing.

The theory of Rabi waves developed by the authors of Refs. 14 and 15 is essentially limited in that it relates to the pure quantum states only. Thus, in the case of Rabi waves in classical light,[14] we have one-particle states described by the wave function $|\Psi\rangle$, which is the coherent superposition of different eigenstates of all QDs in a chain. The excitation and propagation of such Rabi waves need the special conditions. The fulfillment of these conditions can be interpreted as the effect of special potentials in the Schrödinger equation and is taken into account by introducing the corresponding terms into the Hamiltonian. For example, the total reflection of a Rabi wave from the ends of a QD chain can be described on an intuitive level by placing the chain into the infinitely deep potential wall.

De facto, the excitation and dissipation of Rabi waves is a consequence of the QD-chain interaction with another quantum system or reservoir which mixes quantum states corresponding to different subsystems. To describe such an interaction one needs to apply the density matrix formalism.[16] Moreover, in the general case the physically justified statement of the initial-value, the boundary-value, and the initial-boundary problems is only possible in terms of the density matrix. It should be noted that the mixing is not only a

 ©2012 American Physical Society

disturbing factor, it opens some interesting possibilities in quantum information processing due to the formation of statistically indefinite states analogous to classical statistical physics. The theoretical study of mixed states of Rabi waves is the subject of the current work.

In this paper we consider a prospective application of Rabi waves—quantum nanoantennas. An antenna is the device transforming the near-zone electromagnetic field into a far-zone field and vice versa.[17] The efficiency of such a transformation strongly depends on the ratio between light wavelength in the antenna and the antenna length, and becomes highest when these quantities are comparable. Recent technological developments made feasible nanosized antennas for terahertz, infrared, and optical ranges, the so-called nanoantennas.[18] Surface plasmons in noble metals and in carbon nanotubes have been proposed for the realization of differently configured antennas, such as bow-tie and single wire and Yagi-Uda vibrator antennas, phase array antennas, and so on.[18–28]

A general property of all types of antennas mentioned above is their classical nature utilizing the excitation of the electromagnetic field by an external source of light by analogy with macroscopic microwave and RF engineering.[17] Meanwhile, other possibilities for the antenna design arise at the nanoscale due to quantum mechanisms of light emission, which have no classical analogs. One such mechanism, the spontaneous emission of excited atoms, has been presented in Ref. 21. Another one, the Rabi oscillations, is considered in our paper. To obtain a significant retardation of field in antenna, which is a compulsory condition of the antenna performance, one should create a spatially distributed system of Rabi oscillators (i.e., a pass to the regime of space propagation of Rabi waves).

The paper is organized as follows. In Sec. II the model and corresponding system of Bloch equations are formulated. In Sec. III eigenmodes of the statistical operator are considered, the general solution of the initial problem is obtained, and the frequency spectra of induced currents of a different nature are investigated. In Sec. IV we apply the theory of mixed states of Rabi waves to the concept of quantum terahertz nanoantennas. The main results of the work are formulated in Sec. V.

II. BLOCH EQUATIONS

A. Model and equations of motion

Let an infinite periodical one-dimensional (1D) chain of identical two-level QDs be aligned with the period a parallel to the x axis. Let $|a_p\rangle$ and $|b_p\rangle$ be one-electron orbital wave functions of the pth QD in the excited and ground states, respectively (Fig. 1). The interlevel transition in pth QD is characterized by the transition frequency ω_0 and the dipole moment $\boldsymbol{\mu}$. Further, we assume that all transition dipole moments in the chain possess the same real value and a fixed orientation. Let two neighboring QDs be coupled through the electron tunneling such that only intraband tunnel transitions are permitted. In parallel with the tunneling, the interdot dipole-dipole interaction also takes place. However, as shown in Ref. 15, tunneling is the dominant mechanism of interdot coupling for a wide realistic range of parameters. Therefore, in the given work we restrict ourselves to accounting for tunnel

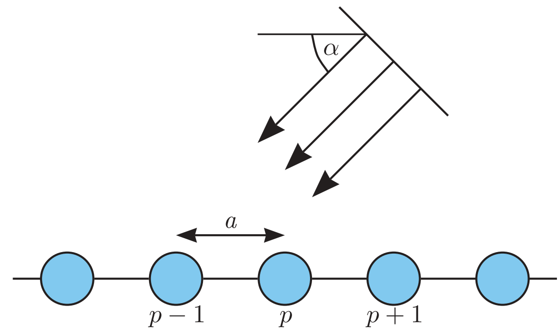

FIG. 1. (Color online) Schematic picture of the QD chain interacting with single-mode electromagnetic field.

transitions only and neglect other mechanisms of interdot coupling.

Let the QD chain be exposed to classical light with time-harmonic electric field [$\exp(-i\omega t)$ time dependence is implicit, ω denotes the angular frequency]. The field dependence on the x coordinate is assumed in the form of a traveling wave with the slowly varying amplitude $\mathbf{E}_0(x)$: $\mathbf{E}(x) = \mathbf{E}_0(x)\exp(ikx)$, where k is the x component of the wave vector. Among a variety of possible external field orientations there are two cases of special interest. The first one is the oblique incidence of the plane wave characterized by $k = (\omega/c)\cos\alpha$, where α is the angle of incidence. The second case is the QD-chain interaction with a strongly slowed-down surface wave ($k \gg \omega/c$). The field dependence on the transverse coordinates is further neglected because QD is an electrically small object (i.e., its spatial extension is much less than the wavelength).

In the rotating wave approximation, the Hamiltonian of the QD chain in the external electric field reads $\hat{H} = \hat{H}_0 + \hat{H}_T$, where the term

$$\hat{H}_0 = \frac{\hbar\omega_0}{2}\sum_p \hat{\sigma}_{zp} - \frac{\hbar}{2}\sum_p [\Omega_{Rp}\hat{\sigma}_p^\dagger e^{ikpa}e^{-i\omega t} + \text{H.c.}] \tag{1}$$

describes the system in the absence of electron tunneling. Here $\hat{\sigma}_{zp} = |a_p\rangle\langle a_p| - |b_p\rangle\langle b_p|$, $\hat{\sigma}_p^+ = |a_p\rangle\langle b_p|$, and $\hat{\sigma}_p^- = |b_p\rangle\langle a_p|$ are the raising and lowering operators of the pth QD, respectively, $\Omega_{Rp} = -\boldsymbol{\mu}\boldsymbol{E}_0(pa)/\hbar$ is the Rabi frequency of the pth QD. The term

$$\begin{aligned}\hat{H}_T = &-\hbar\xi_1\sum_p (|a_p\rangle\langle a_{p+1}| + |a_p\rangle\langle a_{p-1}|)\\ &-\hbar\xi_2\sum_p (|b_p\rangle\langle b_{p+1}| + |b_p\rangle\langle b_{p-1}|)\end{aligned} \tag{2}$$

accounts for the interdot electron tunneling. Here $\xi_{1,2}$ are the electron tunneling frequencies for the excited (ξ_1) and ground (ξ_2) states of the QD. The tunneling frequencies $\xi_{1,2}$ and the dipole moment $\boldsymbol{\mu}$ enter further analysis as phenomenological parameters which are assumed to be *a priori* known. Realistic values of these parameters have been estimated by the authors of Ref. 15 using both theoretical and experimental data.

The total density operator $\hat{\rho}_{\text{tot}}$ describing the QD-chain dynamics in the exposing electromagnetic field can be divided into 2×2 blocks:

$$\hat{\rho}_{pq} = \begin{pmatrix} \rho_{pq}^{aa} & \rho_{pq}^{ab} \\ \rho_{pq}^{ba} & \rho_{pq}^{bb} \end{pmatrix}. \tag{3}$$

The diagonal elements $\rho_{pq}^{aa,bb}$ of these blocks at $p = q$ are probabilities of the ground and excited states in the pth QD. All the other elements describe the quantum correlations of the states which are governed by the interdot coupling and QD-light interaction.

The evolution of the system is described by the Liouville equation $\partial\hat{\rho}_{\rm tot}/\partial t = -i[\hat{H},\hat{\rho}_{\rm tot}]/\hbar$, where the Hamiltonian is given by Eqs. (1) and (2). This Liouville equation leads to the following recurrent equations for 2×2 blocks:

$$\begin{aligned}\frac{\partial\rho_{pq}^{aa}}{\partial t} &= \frac{i}{2}\left[\Omega_{Rp}\rho_{pq}^{ba}e^{i(kpa-\omega t)} - \Omega_{Rq}^{*}\rho_{pq}^{ab}e^{-i(kqa-\omega t)}\right] \\ &\quad + i\xi_1\left(\rho_{p-1,q}^{aa} + \rho_{p+1,q}^{aa} - \rho_{p,q-1}^{aa} - \rho_{p,q+1}^{aa}\right),\end{aligned} \tag{4}$$

$$\begin{aligned}\frac{\partial\rho_{pq}^{bb}}{\partial t} &= \frac{i}{2}\left[\Omega_{Rp}^{*}\rho_{pq}^{ab}e^{-i(kpa-\omega t)} - \Omega_{Rq}\rho_{pq}^{ba}e^{i(kqa-\omega t)}\right] \\ &\quad + i\xi_2\left(\rho_{p-1,q}^{bb} + \rho_{p+1,q}^{bb} - \rho_{p,q-1}^{bb} - \rho_{p,q+1}^{bb}\right),\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{\partial\rho_{pq}^{ab}}{\partial t} &= -i\omega_0\rho_{pq}^{ab} + \frac{i}{2}\left(\Omega_{Rp}\rho_{pq}^{bb}e^{ikpa} - \Omega_{Rq}\rho_{pq}^{aa}e^{ikqa}\right)e^{-i\omega t} \\ &\quad + i\xi_1\left(\rho_{p-1,q}^{ab} + \rho_{p+1,q}^{ab}\right) - i\xi_2\left(\rho_{p,q-1}^{ab} + \rho_{p,q+1}^{ab}\right).\end{aligned} \tag{6}$$

B. Continuous limit transition

Equations of motion given by the recurrent chain of ordinary differential equations (4) through (6) can be reduced to a more convenient presentation in the form of a system of partial differential equations. To do that, we pass to the continuous limit as has been done by the authors of Ref. 15 [i.e., we substitute $pa \to x$, $qa \to x'$, $\rho_{pq}^{ab} \to \rho_{ab}(x,x',t)$, $\rho_{p-1,q}^{ab} + \rho_{p+1,q}^{ab} - 2\rho_{p,q}^{ab} \to a^2\partial^2\rho_{ab}/\partial x^2$, $\rho_{p,q-1}^{ab} + \rho_{p,q+1}^{ab} - 2\rho_{p,q}^{ab} \to a^2\partial^2\rho_{ab}/\partial x'^2$, $\Omega_{Rp} \to \Omega_R(x)$, $\Omega_{Rq} \to \Omega_R(x')$]. As a result, from Eqs. (4) to (5) we obtain the system of partial differential equations as follows:

$$\begin{aligned}\frac{\partial\tilde{\rho}_{aa}}{\partial t} &= \frac{i}{2}[\Omega_R(x)\tilde{\rho}_{ba} - \Omega_R^{*}(x')\tilde{\rho}_{ab}] + i\xi_1 a^2 \\ &\quad \times \left(\frac{\partial^2\tilde{\rho}_{aa}}{\partial x^2} - \frac{\partial^2\tilde{\rho}_{aa}}{\partial x'^2}\right) - \xi_1 k a^2\left(\frac{\partial\tilde{\rho}_{aa}}{\partial x} + \frac{\partial\tilde{\rho}_{aa}}{\partial x'}\right),\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{\partial\tilde{\rho}_{bb}}{\partial t} &= \frac{i}{2}[\Omega_R^{*}(x)\tilde{\rho}_{ab} - \Omega_R(x')\tilde{\rho}_{ba}] + i\xi_2 a^2 \\ &\quad \times \left(\frac{\partial^2\tilde{\rho}_{bb}}{\partial x^2} - \frac{\partial^2\tilde{\rho}_{bb}}{\partial x'^2}\right) + \xi_2 k a^2\left(\frac{\partial\tilde{\rho}_{bb}}{\partial x} + \frac{\partial\tilde{\rho}_{bb}}{\partial x'}\right),\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\partial\tilde{\rho}_{ab}}{\partial t} &= -i\Delta\tilde{\rho}_{ab} + \frac{i}{2}[\Omega_R(x)\tilde{\rho}_{bb} - \Omega_R(x')\tilde{\rho}_{aa}] \\ &\quad - \xi_1 k a^2\frac{\partial\tilde{\rho}_{ab}}{\partial x} + \xi_2 k a^2\frac{\partial\tilde{\rho}_{ab}}{\partial x'} + i\xi_1 a^2\frac{\partial^2\tilde{\rho}_{ab}}{\partial x^2} \\ &\quad - i\xi_2 a^2\frac{\partial^2\tilde{\rho}_{ab}}{\partial x'^2},\end{aligned} \tag{9}$$

where $\tilde{\rho}_{aa} = \rho_{aa}\exp\{-ik(x-x')/2\}$, $\tilde{\rho}_{bb} = \rho_{bb}\exp\{ik(x-x')/2\}$, $\tilde{\rho}_{ab} = \rho_{ab}\exp\{i\omega t\}\exp\{-ik(x+x')/2\}$, $\tilde{\rho}_{ba} = \rho_{ba}\exp\{-i\omega t\}\exp\{ik(x+x')/2\}$, $\Delta = \omega_0 - \omega + (\xi_1 - \xi_2)(2 + a^2k^2/4)$.

Taking the complex conjugation of Eq. (9) and changing $x \to x'$, $x' \to x$, we obtain

$$\begin{aligned}\frac{\partial\tilde{\rho}_{ba}}{\partial t} &= i\Delta\tilde{\rho}_{ba} - \frac{i}{2}[\Omega_R(x')\tilde{\rho}_{bb} - \Omega_R(x)\tilde{\rho}_{aa}] - \xi_1 k a^2\frac{\partial\tilde{\rho}_{ba}}{\partial x'} \\ &\quad + \xi_2 k a^2\frac{\partial\tilde{\rho}_{ba}}{\partial x} - i\xi_1 a^2\frac{\partial\tilde{\rho}_{ba}}{\partial x'^2} + i\xi_2 a^2\frac{\partial\tilde{\rho}_{ba}}{\partial x^2}.\end{aligned} \tag{10}$$

The normalization condition $\mathrm{Tr}(\hat{\rho}_{\rm tot}) = \sum_p \mathrm{Tr}(\hat{\rho}_{pp}) = 1$ is transformed to

$$\frac{1}{a}\int_{-\infty}^{\infty}\rho_{aa}(x,x)dx + \frac{1}{a}\int_{-\infty}^{\infty}\rho_{bb}(x,x)dx = 1. \tag{11}$$

Equations (7) through (10) with the normalization condition (11) constitute the generalization of the Bloch equations for the case of a spatially distributed collection of coupled two-level systems.

C. Master equations

To facilitate further analysis we rewrite the system (7) through (10) in the compact matrix form with respect to the 2×2 effective density matrix $\hat{\rho} = \left(\begin{smallmatrix}\tilde{\rho}_{bb} & \tilde{\rho}_{ba} \\ \tilde{\rho}_{ab} & \tilde{\rho}_{aa}\end{smallmatrix}\right)$. It reads

$$\frac{\partial\hat{\rho}}{\partial t} = -\frac{i}{\hbar}[\hat{H}_{\rm eff},\hat{\rho}] - \frac{1}{2}\{\hat{\Gamma},\hat{\rho}\}, \tag{12}$$

where $\{\ldots,\ldots\}$ denotes the anticommutator of two matrices. Equation (12) represents the master equation in typical form (see, for example, Ref. 10) with the effective Hamiltonian

$$\hat{H}_{\rm eff} = \frac{\hbar}{2}\begin{pmatrix} -\tilde{\Delta} & -\tilde{\Omega}_R \\ -\tilde{\Omega}_R & \tilde{\Delta} \end{pmatrix}, \tag{13}$$

where

$$\begin{aligned}\tilde{\Delta} &= \Delta + (\xi_2 - \xi_1)\frac{a^2}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x'^2}\right) \\ &\quad - i(\xi_2 + \xi_1)\frac{ka^2}{2}\left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right),\end{aligned} \tag{14}$$

$$\tilde{\Omega}_R = \frac{\Omega_R(x) + \Omega_R(x')}{2}. \tag{15}$$

The effective dissipation operator reads as follows:

$$\hat{\Gamma} = -\begin{pmatrix} \hat{\Gamma}_{bb} & \Delta\Omega \\ \Delta\Omega & \hat{\Gamma}_{aa} \end{pmatrix}, \tag{16}$$

where

$$\Delta\Omega = \frac{\Omega_R(x') - \Omega_R(x)}{2}, \tag{17}$$

$$\hat{\Gamma}_{bb} = i\xi_2 a^2\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial x'^2}\right) + \xi_2 a^2 k\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial x'}\right), \tag{18}$$

$$\hat{\Gamma}_{aa} = i\xi_1 a^2\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial x'^2}\right) - \xi_1 a^2 k\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial x'}\right). \tag{19}$$

The matrix elements of the effective Hamiltonian and effective dissipation operator are expressed in terms of partial differential operators acting element-by-element on matrix $\hat{\rho}$ because of its dependence on the QD coordinates x,x'. It should be noted that in the case of the inhomogeneous electric field the coefficients of Eq. (12) also include the QD coordinates x,x'. It can easily be verified that operators $\hat{H}_{\text{eff}}$ and $\hat{\Gamma}$ are Hermitian because

$$\hat{H}_{\text{eff},\alpha\beta}(x,x') = \hat{H}^{*}_{\text{eff},\beta\alpha}(x',x), \tag{20}$$

$$\hat{\Gamma}_{\alpha\beta}(x,x') = \hat{\Gamma}^{*}_{\beta\alpha}(x',x), \tag{21}$$

where $\alpha,\beta = a,b$. Note that operators $\hat{H}_{\text{eff}}$ and $\hat{\Gamma}$ do not commutate.

The equation of motion (12) can be represented in the superoperator Lindblad-like form as $\partial\hat{\rho}/\partial t = L\hat{\rho}$, where the linear superoperator

$$L = -\frac{i}{\hbar}[\hat{H}_{\text{eff}},\ldots] - \frac{1}{2}\{\hat{\Gamma},\ldots\} \tag{22}$$

maps the operator $\hat{\rho}(x,x',0)$ into another operator $\hat{\rho}(x,x',t)$.

It is not surprising that Eqs. (7) to (10) can be rewritten in the Lindblad-like form. Indeed, these equations are written for the single-particle density matrix. Lindblad forms always appear in equations of motion of open quantum systems and describe the interaction of such systems with quantum reservoirs of a different nature, which results in dissipation.[10] A single QD in the chain can be considered as an open quantum system and the rest of the chain as the quantum reservoir. In this case the tunneling is the interaction giving rise to Lindblad-like components [second term in the right-hand part of Eq. (12)]. Because this term results from the exciton diffusion from QD and is not related to the dissipation, the Lindblad-like form involves the spatial differential operators (18) and (19).

Equation (12) with coefficients given by Eqs. (13) to (15) is the basis system for our analysis. This system describes the wave motion of quantum transitions along the QD chain. In our formulation, this process is characterized by the matrix wave field $\hat{\rho}$ and dictated by the interplay of Rabi oscillations in the external electromagnetic field and interdot tunneling. For a one-dimensional QD chain this wave process is developed in the three-dimensional (3D) space-time $(\mathbf{r},t)$ (one temporal variable t and two spatial variables x and x'), where $\mathbf{r} = \mathbf{e}x + \mathbf{e}'x'$, $\mathbf{e}$, $\mathbf{e}'$ are corresponding orthogonal unit vectors. Note that Eq. (12) may not guarantee the correspondence of its formal mathematical solution to the real physical state. In order for operator $\hat{\rho}(x,x',t)$ to really represent the statistical operator of a quantum system, three fundamental properties should be preserved: (i) The operator must be Hermitian; (ii) The operator must satisfy the conservation law; (iii) The operator must be positive in the Hilbert space, that is, the condition

$$\iint \langle\psi(x')|\hat{\rho}(x,x',t)|\psi(x)\rangle dx dx' \geqslant 0 \tag{23}$$

must hold true for any $|\psi(x')\rangle$ at any t. The hermiticity of the operator $\hat{\rho}$ requires $\hat{\rho}_{\alpha\beta}(x,x',t) = \hat{\rho}_{\beta\alpha}(x',x,t)$ and follows immediately from the hermiticity of operators $\hat{H}_{\text{eff}}$ and $\hat{\Gamma}$ [see Eqs. (20) and (21)]. It can easily be shown that Eq.(12) preserves property (ii).

In some cases master equations do not guarantee property (iii).[29] The positivity of the statistical operator may be lost for specific times and also depends on the initial conditions, thus one needs to check if condition (iii) is indeed fulfilled.[29] Taking into account hermiticity conditions (20) and (21) one can demonstrate by direct substitution that the solution of Eq. (12) satisfies

$$\hat{\rho}(x,x',t) = \hat{S}^{\dagger}\hat{\rho}(x,x',t_0)\hat{S}, \tag{24}$$

where $\hat{S}$ is the *nonunitary* evolution operator. This operator can be written as

$$\hat{S} = \hat{S}(x,x',t,t_0) = \exp\left(\frac{i}{\hbar}\hat{H}_{\text{eff}} - \frac{1}{2}\hat{\Gamma}\right)(t-t_0) \tag{25}$$

(the dependence of $\hat{H}_{\text{eff}}$ and $\hat{\Gamma}$ on x, x' is omitted for brevity). The expressions for $\hat{S}^{\dagger}$ can be obtained from Eq. (25) by substituting $i \to -i$. From Eq. (24) it follows that if the nonnegativity of the statistical operator (12) take place for $t = t_0$ then this property holds true for any $t > t_0$ also. Thus, Eq. (12) guarantees the fulfillment of condition (iii) for any initial condition satisfying condition (iii).

To simplify our theoretical analysis, we consider first the finite-length QD chain with periodic boundary conditions at the chain ends. It corresponds to the definition of the normalization area for quantum field $\hat{\rho}(\mathbf{r},\mathbf{t})$ in the form of a square with the size l, where l is the length of the QD chain. Equation (12) should be solved inside the normalization area with the boundary conditions $\hat{\rho}(-l/2,x',t) = \hat{\rho}(l/2,x',t)$ and $\hat{\rho}(x,-l/2,t) = \hat{\rho}(x,l/2,t)$ for $t \geqslant 0$ (the Born–von Karman boundary condition). In agreement with the standard requirement of quantum field theory,[10] in the limit $l \to \infty$ we arrive at the case of free propagation in infinite space.

D. Observable values

Let us characterize the spatial-temporal dynamics of Rabi oscillations in the QD chain by the spatial density of the inversion (the inversion per single QD)

$$w(x,t) = a[\rho_{aa}(x,x,t) - \rho_{bb}(x,x,t)]. \tag{26}$$

Another important quantity is the current per unit cell $J(x)$ which can be decomposed into a polarization part J_{pol} and a tunneling part J_{tun} as follows:

$$J(x) = J_{\text{pol}}(x) + J_{\text{tun}}(x). \tag{27}$$

The current components $J_{\text{pol}}(x)$, $J_{\text{tun}}(x)$ are related to the density matrix $\hat{\rho}$ by the expressions[29]

$$J_{\text{tun}}(x) = ia^2 e \lim_{x\to x'}\left[\xi_1\frac{\partial\rho_{aa}(x,x',t)}{\partial x} + \xi_2\frac{\partial\rho_{bb}(x,x',t)}{\partial x}\right] + \text{c.c.}, \tag{28}$$

$$J_{\text{pol}}(x) = -i\omega[(\mathbf{e}_x\boldsymbol{\mu})\rho_{ba}(x,x,t)e^{i(kx-\omega t)} - \text{c.c.}], \tag{29}$$

where e are the free electron charge, V is the QD volume (it sets the natural lower bound for interdot distance $a \sim \sqrt[3]{V}$).

Each of two current components represents its own type of carrier motion. The tunneling current (28) corresponds to the intralevel drift of the particle along the chain. It is comprised of partial contributions of the ground state ($\sim\xi_2$) and excited state ($\sim\xi_1$) particles. The polarization current (29) corresponds to the quantum jumps of the particles between the ground and excited states, dictated by the exposing electric field.

E. Equations of motion for spatially homogeneous Rabi waves

Let us move to the spatially homogeneous limit of equations of motion (12). Let the QD chain be exposed to be a homogeneous plane wave which corresponds to the spatially independent Rabi frequency $\tilde{\Omega}_R(x,x') = \text{const}$, $\Delta\Omega = 0$. We shall consider the spatially homogeneous quantum states in the QD chain described by the density matrix dependent only on the difference of spatial variables $\hat{\rho}(x,x',t) = \hat{\rho}(x - x',t)$. In this partial case the general equations (12) can be simplified because the wave process under examination is developed in the two-dimensional (2D) space-time $(x - x',t)$. The spatial homogeneity leads to the symmetry of the density matrix

$$\frac{\partial\hat{\rho}}{\partial x} = -\frac{\partial\hat{\rho}}{\partial x'}. \tag{30}$$

Substituting Eq. (30) into the general system of Bloch equations we can replace $\hat{\rho}(x - x',t) \to \hat{\rho}(x,t)$ and obtain the equations of motion for $\hat{\rho}(x,t)$ in the form (12) with operators $\hat{H}_{\text{eff}}$ and $\hat{\Gamma}$ reduced to

$$\hat{H}_{\text{eff}} = \frac{\hbar}{2}\begin{pmatrix} -\tilde{\Delta} & -\tilde{\Omega}_R \\ -\tilde{\Omega}_R & \tilde{\Delta} \end{pmatrix}, \tag{31}$$

$$\hat{\Gamma} = -\begin{pmatrix} \hat{\Gamma}_{bb} & 0 \\ 0 & \hat{\Gamma}_{aa} \end{pmatrix}, \tag{32}$$

where $\tilde{\Delta} = \Delta + (\xi_2 - \xi_1)a^2\partial^2/\partial x^2$, $\hat{\Gamma}_{aa,bb} = \mp 2\xi_{1,2}ka^2\partial/\partial x$. Equations (28) and (29) for observable currents are reduced to the form

$$J_{\text{tun}} = ia^2e\lim_{x\to 0}\left[\xi_1\frac{\partial\rho_{aa}(x,t)}{\partial x} + \xi_2\frac{\partial\rho_{bb}(x,t)}{\partial x}\right] + \text{c.c.}, \tag{33}$$

$$J_{\text{pol}}(x) = -i\omega[(\mathbf{e}_x\boldsymbol{\mu})\rho_{ba}(0,t)e^{i(kx-\omega t)} - \text{c.c.}]. \tag{34}$$

From Eqs. (33) and (34) one can see that in the homogeneous case the tunneling current becomes independent of the QD coordinate x while the polarization current depends on x according to the traveling wave law for electric field $\exp(ikx)$. Such a behavior is physically justified.

III. GENERAL SOLUTION OF INITIAL-VALUE PROBLEM FOR STATISTICAL OPERATOR

A. Eigenmodes of statistical operator

Let us find elementary solutions of the system (12) in the form of traveling waves for the case of a Rabi frequency homogeneous along the chain axis. We present the solution in the form of

$$\hat{\rho}_{\mathbf{k}}(\mathbf{r},t) = \tfrac{1}{2}\left(\hat{\rho}^0_{\mathbf{k}}(\mathbf{r})e^{i\nu(\mathbf{k})t} + \text{H.c.}\right), \tag{35}$$

where ν and $\hat{\rho}^0_{\mathbf{k}}(\mathbf{r})$ are the eigenfrequency and eigenstate to be found of the superoperator L. For these values Eq. (12) leads to the eigenvalue problem[29]

$$L\hat{\rho}^0_{\mathbf{k}} = i\nu(\mathbf{k})\hat{\rho}^0_{\mathbf{k}}. \tag{36}$$

The eigenstate $\hat{\rho}^0_{\mathbf{k}}$ can be represented in terms of the normalized plane waves $\varphi_{\mathbf{k}}(\mathbf{r}) = l^{-1}e^{i\mathbf{k}\mathbf{r}}$ with the momentum $\mathbf{k} = (\mathbf{e}h - \mathbf{e}'\kappa) = 2\pi(\mathbf{e}m - \mathbf{e}'n)/l$ (m and n are arbitrary integers) as

$$\hat{\rho}^0_{\mathbf{k}} = \hat{\tau}_{\mathbf{k}}\varphi_{\mathbf{k}}(\mathbf{r}), \tag{37}$$

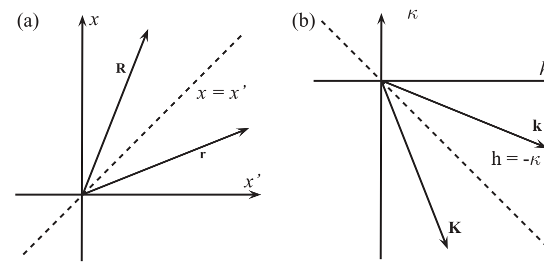

FIG. 2. Coordinate systems used in the consideration of the mixed states of Rabi waves: (a) geometrical space, $\mathbf{r} = \mathbf{e}x + \mathbf{e}'x'$, $\mathbf{R} = \mathbf{e}'x + \mathbf{e}x'$; (b) phase space, $\mathbf{k} = \mathbf{e}h - \mathbf{e}'\kappa$, $\mathbf{K} = -\mathbf{e}'h + \mathbf{e}\kappa$.

where $\hat{\tau}_{\mathbf{k}}$ is a constant 2×2 matrix. Note that the chosen ansatz (35) in view of Eq. (37) automatically satisfies the physically obligatory relation of symmetry for the statistical operator $\hat{\rho}_{\mathbf{k}}(\mathbf{r},t) = \hat{\rho}^{\dagger}_{\mathbf{k}}(\mathbf{R},t)$, where $\mathbf{R}$ is shown in Fig. 2(a). The chosen configuration of the Brillouin zone in the $\mathbf{k}$ space [Fig. 2(b)] represents the half-plane $h > -\kappa$. The opposite half-plane is further eliminated because it does not contain any new quantum states.

Substituting Eq. (37) into Eq. (36) and omitting the common factors, we obtain the homogeneous supermatrix equation. The existence of a nontrivial solution with respect to matrix $\hat{\tau}_{\mathbf{k}}$ requires the respective determinant to be zero. It leads to a fourth-order equation, which yields the values $\nu(\mathbf{k})$. Solving it with respect to ν, we determine the eigenfrequencies of the system as

$$\nu_{1,2}(\mathbf{k}) = \Delta_1 + \sqrt{\frac{\Omega^2 \pm \sqrt{\Omega^4 - 4\Delta_2^2\Delta_4^2}}{2}}, \tag{38}$$

$$\nu_{3,4}(\mathbf{k}) = \Delta_1 - \sqrt{\frac{\Omega^2 \pm \sqrt{\Omega^4 - 4\Delta_2^2\Delta_4^2}}{2}}, \tag{39}$$

for $m^2 > n^2$ [or $(\mathbf{ke})^2 > (\mathbf{ke}')^2$].Here the sign "+" under the root corresponds to $\nu_{1,3}$ and the sign "−" to $\nu_{2,4}$. In the foregoing equations $\Omega^2 = \Omega_R^2 + \Delta_2^2 + \Delta_4^2$,

$$\Delta_1 = \frac{a^2}{2}[(h^2 - \kappa^2)(\xi_1 + \xi_2) - k(h - \kappa)(\xi_2 - \xi_1)], \tag{40}$$

$$\Delta_2 = \frac{a^2}{2}[(h^2 - \kappa^2)(\xi_2 - \xi_1) - k(h - \kappa)(\xi_1 + \xi_2)], \tag{41}$$

$$\Delta_4 = \Delta - \frac{a^2}{2}(\xi_2 - \xi_1)(h^2 + \kappa^2) - \frac{ka^2}{2}(\xi_2 + \xi_1)(h + \kappa). \tag{42}$$

The lines $h = \kappa$ and, correspondingly, the points $m = n$ form an infinite set of branching points for functions (38) and (39). Therefore, for $m^2 < n^2$ [or $(\mathbf{ke})^2 < (\mathbf{ke}')^2$] the correct choice of the branch requires the signs before the first square roots in Eqs. (38) and (39) to be changed to opposite ones. One can see that eigenvalues $\nu_j(\mathbf{k})$ satisfy the equality $\nu_j(\mathbf{K}) = -\nu_j(\mathbf{k})$.

The corresponding matrices $\hat{\tau}_{j\mathbf{k}}$ are defined by the expressions

$$\hat{\tau}_{j\mathbf{k}} = \frac{C_{j\mathbf{k}}}{2}\begin{pmatrix} \frac{\nu_j - \Delta_1 + \Delta_2}{\nu_j - \Delta_1} & \frac{\Omega_R}{\nu_j - \Delta_1 + \Delta_4} \\ -\frac{\Omega_R}{\nu_j - \Delta_1 - \Delta_4} & -\frac{\nu_j - \Delta_1 - \Delta_2}{\nu_j - \Delta_1} \end{pmatrix}, \tag{43}$$

where $C_{j\mathbf{k}}$ are the normalization coefficients with $j = 1,2,3,4$, ν_j are defined by Eqs. (38) and (39) (for short, we omit the dependence on $\mathbf{k}$ in $\nu_j, \Delta_1, \Delta_2, \Delta_4$). One can see that matrices (43) possess the symmetry property $\hat{\tau}_{j\mathbf{k}} = \hat{\tau}^{\dagger}_{j\mathbf{K}}$.

Note that the set of eigenstates of superoperators given by Eq. (37) forms an orthogonal basis and the normalization constants $C_{j\mathbf{k}}$ are chosen to satisfy the scalar orthonormality relation

$$\iint \mathrm{Tr}\{\hat{\rho}^0_{i\mathbf{k}}(\mathbf{r})[\hat{\rho}^0_{j\mathbf{k}'}(\mathbf{r})]^{\dagger}\} d^2\mathbf{r} = \delta_{ij}\delta(\mathbf{k}-\mathbf{k}'). \tag{44}$$

Let us consider Eqs. (38) and (39). In the limit of spatial uniformity $h = \kappa$ we have $\Delta_{1,2} \to 0$, which reduces Eqs. (38) and (39) to $\nu_{1,3} \to \pm\Omega$ and $\nu_{2,4} \to 0$. Thus, one can conclude that the eigenmodes with $j = 2,4$ represent the steady states of the system. The corresponding eigenmatrixes are defined by the expressions

$$\hat{\tau}_{2,4\mathbf{k}} = \frac{1}{2\Delta_4}\begin{pmatrix} \Omega + \Delta_4 & \Omega_R \\ \Omega_R & \Omega - \Delta_4 \end{pmatrix}. \tag{45}$$

Because of $\hat{\tau}^2_{2,4\mathbf{k}} = C^2\hat{\tau}_{2,4\mathbf{k}}$, where $C = \Omega_R(1+p_1^2)/(2p_1\Delta_4)$, $p_1 = \Omega_R/(\Delta_4 + \sqrt{\Omega_R^2 + \Delta_4^2})$, the above matrices describe the pure quantum states. These modes represent the coherent superposition of the ground and excited states of the chain exposed to an electromagnetic field and can be associated with the rabitons introduced by the authors of Ref. 15.

For $j = 1,3$ Eq. (43) amounts to the following expression:

$$\lim_{h\to\kappa} \hat{\tau}_{1,3\mathbf{k}} \to \frac{1}{2}\begin{pmatrix} 1 & \frac{\Omega_R}{\nu_{1,3}+\Delta_4} \\ -\frac{\Omega_R}{\nu_{1,3}-\Delta_4} & -1 \end{pmatrix}. \tag{46}$$

One can see that Eq. (46) represents the unphysical solution because it contradicts the property of positivity. It means that this solution (as well as some other eigenstates of superoperator L) does not correspond to a real physical state. But these states cannot be discarded because they are elements of the complete basis of states and therefore appear as partial terms of the general state decomposition in this basis.

B. Solution of the initial problem

Consider a general solution of Eq. (12) with the initial condition

$$\hat{\rho}(\mathbf{r},t)\,|_{t=0} = \hat{\rho}(\mathbf{r},0). \tag{47}$$

Further we assume that the initial matrix $\hat{\rho}(\mathbf{r},0)$ satisfies the conditions of physical realizability, that is, this matrix is Hermitian, positive-definite, and $\mathrm{Tr}\{\hat{\rho}(\mathbf{r},0)\} = 1$.

It is easy to show that the positive-definiteness condition requires the initial state to be an arbitrary superposition of the rabitons (i.e., states with $j = 2,4$). The positive-definiteness can also be achieved if Eq. (47) includes states (46). In this case Eq. (47) should contain at least one rabiton and amplitudes of the states (46) should be small enough [it comports with the statement that states (46) do not exist separately but only in different superpositions]. This type of initial condition is of the most interest because the presence of state (46) leads to the qualitatively new effects. One of them (low-frequency tunnel current) will be considered below as the basis for new types of nanoantennas.

For the general solution of Eq. (12) we shall seek an expansion with respect to eigenfunctions of the statistical operator determined by Eqs. (37) through (39) and Eq. (43):

$$\hat{\rho}(\mathbf{r},t) = \sum_{i=1}^{4}\sum_{\mathbf{k}}\int_{-\infty}^{\infty} S^i_{\mathbf{k},\nu}\rho^0_{i,\mathbf{k}} e^{-i\nu t}\, d\nu\,, \tag{48}$$

where $S^i_{\mathbf{k},\nu}$ are the coefficients to be found. These coefficients can be evaluated by substituting Eq. (48) into Eq. (12) and making use of the scalar orthogonality condition (44). Substituting Eq. (48) into Eq. (12), one should take into account that the differentiation over t in the integral in Eq. (48) must be performed by integration by parts. After such manipulations we come to

$$S^i_{\mathbf{k},\nu} = -\frac{2i}{\nu - \nu_i(\mathbf{k})}\mathrm{Tr}(\hat{f}_{\mathbf{k}}\hat{\tau}^{\dagger}_{i\mathbf{k}})\,, \tag{49}$$

where

$$\hat{f}_{\mathbf{k}} = \int \hat{\rho}(\mathbf{r},0)\varphi^*_{\mathbf{k}}(\mathbf{r})\, d^2\mathbf{r}\,. \tag{50}$$

Relations (48) to (50) define the general solution of the initial problem. Let us substitute Eq. (49) into Eq. (48). We obtain

$$\hat{\rho}(\mathbf{r},t) = -2i\sum_{i=1}^{4}\sum_{\mathbf{k}}\int_{-\infty}^{\infty}\frac{\mathrm{Tr}(\hat{f}_{\mathbf{k}}\hat{\tau}^{\dagger}_{i\mathbf{k}})\rho^0_{i,\mathbf{k}}}{\nu - \nu_i(\mathbf{k}) + i0}e^{-i\nu t}\, d\nu, \tag{51}$$

where the symbol $i0$ defines the standard[30] rule for the poles tracing under integration over ν.

The integration into Eq. (51) is taken over all phase space. It is more convenient (particularly, for the analysis of the spatially homogeneous case, see below) to divide the integration area into three zones: $\mathbf{k}$ zone with the wave vector corresponding to the condition $h > -\kappa$, $\mathbf{K}$ zone with the wave vector corresponding to the condition $h < -\kappa$, and $\mathbf{k}_0$ zone corresponding to the case $h = -\kappa$ [see Fig. 2(b)]. Taking into account that the contributions from the $\mathbf{k}$ and $\mathbf{K}$ zones are equal, we rewrite Eq. (51) in the form of

$$\hat{\rho}(\mathbf{r},t) = -2i\sum_{i=1}^{4}\sum_{\mathbf{k}_0}\int_{-\infty}^{\infty}\frac{\mathrm{Tr}\big(\hat{f}_{\mathbf{k}_0}\hat{\tau}^{\dagger}_{i\mathbf{k}_0}\big)\hat{\tau}_{i\mathbf{k}_0}\varphi_{\mathbf{k}_0}(\mathbf{r})}{\nu - \nu_i(\mathbf{k}_0) + i0}e^{-i\nu t}\, d\nu + \Delta\hat{\rho}(\mathbf{r},t)\,. \tag{52}$$

Here

$$\Delta\hat{\rho}(\mathbf{r},t) = 4\pi\sum_{i=1}^{4}\sum_{\mathbf{k}}\int_{-\infty}^{\infty}\mathrm{Tr}(\hat{f}_{\mathbf{k}}\hat{\tau}^{\dagger}_{i\mathbf{k}}) \times [\hat{\tau}_{i\mathbf{k}}\varphi_{\mathbf{k}}(\mathbf{r})e^{-i\nu_i(\mathbf{k})t} + \hat{\tau}^{\dagger}_{i\mathbf{k}}\varphi^*_{\mathbf{k}}(\mathbf{R})e^{i\nu_i(\mathbf{k})t}] \tag{53}$$

and $\mathbf{R}$ is shown in Fig. 2.

Consider now a particular case of the spatially homogeneous state. For that aim, we assume that the initial condition (47) satisfies the condition of the spatial homogeneity $\hat{\rho}(\mathbf{r},0) = \hat{\rho}(x - x',0)$. Then we perform in Eqs. (52) to (53) the limiting transition to an infinite chain $\sum_{\mathbf{k}_0} \to l\int_{-\infty}^{\infty}\{\cdots\}dh$. In the spatially homogeneous case the term (53) reduces to zero. As a result, we arrive at

$$\hat{\rho}(x,t) = 2l\int_{-\infty}^{\infty}[\mathrm{Tr}(\hat{f}_h\hat{\tau}_{2h}^{\dagger})\hat{\tau}_{2h} + \mathrm{Tr}(\hat{f}_h\hat{\tau}_{4h}^{\dagger})\hat{\tau}_{4h}]\varphi_h(x)dh - \frac{2i}{\pi}\int_{-\infty}^{\infty}e^{-i\nu t}\int_{-\infty}^{\infty}\left[\frac{\mathrm{Tr}(\hat{f}_h\hat{\tau}_{1h})\hat{\tau}_{1h}^{\dagger}\varphi_h(x)}{\nu+\nu_1+i0} + \frac{\mathrm{Tr}(\hat{f}_h\hat{\tau}_{1h}^{\dagger})\hat{\tau}_{1h}\varphi_h(x)}{\nu-\nu_1+i0}\right]dhd\nu\,, \tag{54}$$

where $\nu_1(h) = \Omega = \sqrt{\Omega_R^2 + \Delta_4^2(h)}$. The quantities $\hat{f}_h$, $\hat{\tau}_{ih}$, and $\varphi_h(x)$ are the spatially homogeneous limits of the quantities $\hat{f}_{\mathbf{k}}$, $\hat{\tau}_{i\mathbf{k}}$, and $\varphi_{\mathbf{k}}(\mathbf{r})$, respectively. They are obtained from Eqs. (43) and (50) by the substitution $h = \kappa$ and the replacement $\exp\{i\mathbf{kr}\} \to \exp\{ihx\}$ (here we redefine the variable x: $x - x' \to x$).

It should be noted that, in accordance with Eq. (54), only states in the form of traveling wave e^{ihx} [and, consequently, monochromatic oscillations with frequency $\nu_1(h)$] are pure states. Signals which transfer information are always characterized by the frequency spectrum. It means that the information transfer with Rabi waves needs Rabi signals (i.e., mixed states of Rabi waves). This fact justifies our interest to the problem under consideration.

C. Low-frequency tunnel current

The current components (33) and (34) have different physical natures and contribute to different regions of the frequency spectrum. The current (34) arises from the polarization of single QD due to interlevel transitions. This is an amplitude-modulated oscillation with the frequency of quantum transition ω. Its physical nature corresponds to the Mollow triplet.[3] But the satellites are not harmonic oscillations with frequencies $\omega \pm \Omega$, they are nonuniformly broadened lines. The reason for this is the Rabi frequency dependence on the wave number h. This peculiarity of the spectrum is inherent to the spatial propagation of Rabi oscillations (i.e., the Rabi waves, predicted by the authors of Ref. 15). The tunneling current (33) is determined by quantum transitions between neighboring QDs in the same quantum state and contributes to the low-frequency (as compared to ω) region of the spectrum. This current has no analogs in the standard theory of Rabi oscillations.[3]

Substituting density operator (54) into the expression for tunneling current (33) and integrating the result with respect to ν we obtain $J_{\mathrm{tun}} = J^{(0)} + J^{(\Omega)}$, where

$$J^{(0)} = -a^2e\int_{-\infty}^{\infty}\mathrm{Tr}(\hat{f}_h\hat{\tau}_{2h}^{\dagger})\frac{\Delta_{4h}\zeta_1(h) - \Omega_h\zeta_2(h)}{\Delta_{4h}}dh, \tag{55}$$

and

$$J^{(\Omega)}(t) = a^2e\int_{-\infty}^{\infty}\zeta_1(h)\mathrm{Tr}(\hat{f}_h\hat{\tau}_{1h})e^{i\Omega_h t}dh + \mathrm{c.c.} \tag{56}$$

Here $\Delta_{4h} = \Delta_4(h)$ and $\Omega_h = \Omega(h)$ and

$$\zeta_{1,2}(h) = 2(\xi_1 \mp \xi_2)h - k(\xi_1 \pm \xi_2). \tag{57}$$

The tunneling current J_{tun} has both dc and ac components given by expressions (55) and (56), correspondingly. From Eq. (55) one can see that $J^{(0)}$ does not depend on the spatial coordinate and goes to zero when $k \to 0$. From the physical point of view this means that the dc current only exists if there is a preferred direction, which is established by the direction of light wave propagation along the chain. Note that Eq. (55) has the component $\sim\xi_1$, which corresponds to the propagation of a ground state exciton along the chain. This motion is caused by the interaction of QD charge carriers with light and is not related to Joule heating and is nondissipative. By its nature the motion is analogous to the dissipationless electron transport in the photon-dressed semiconductor nanorings predicted by the authors of Ref. 31. The difference is that the effect predicted by the authors of Ref. 31 demonstrates the quadratic dependency of the current on the field because the electron-photon interaction is assumed to be small and is accounted for in the first nonvanishing order of the perturbation theory. The field dependence of current in Eq. (55) has a more complicated form because we do not restrict ourselves to the smallness of the electron-photon interaction.

The existence of the ac component (56) of the tunneling current originated from the spatial propagation of Rabi oscillations we consider as the main prediction of the current paper. The current has a number of peculiar properties, which we further consider in detail. First of all we should note that $J^{(\Omega)}(t)$ is generally not harmonic. Its frequency spectrum is defined by the spatial structure of the initial state $\hat{\rho}(x,0)$ and is given by $\hat{f}_h$. If the initial state in the homogeneous chain is uniform then $\hat{\rho}(x,0)$ does not depend on x. As a result, $\hat{f}_h \sim \delta(h)$, where $\delta(h)$ is the Dirac delta function, and the ac component of the tunneling current $J^{(\Omega)}(t) \sim k(\xi_1 + \xi_2)$. Next, as has been shown earlier (see, for example, Ref. 15), Rabi oscillations propagate in space only if the light wave propagates along the chain. If the light wave is absent ($k \to 0$), then $J^{(\Omega)}(t) \to 0$ at any form of initial state $\hat{\rho}(x,0)$.

The fulfillment of the afore-stated conditions corresponds to a specific spatial asymmetry of the QD chain interacting with light. This asymmetry emerges due to the preferential direction of the light propagation along the chain and thus exists even in the case of a geometrically symmetric (homogeneous) chain. That is why the current $J^{(\Omega)}(t)$ is, in some sense, similar to the low-frequency component of Rabi oscillations in two-level systems with broken inversion symmetry predicted by the authors of Ref. 32. Indeed, in Ref. 32 it was shown that the space symmetry violation leads to nonzero diagonal matrix elements of dipole moment μ_{aa} and μ_{bb}. The interaction of such a system with light leads to the parametrical oscillations of $|a\rangle$ and $|b\rangle$ states together with periodical quantum transitions between these states. As a result, the polarization proportional to $\mu\mathbf{e}(\mu_{aa} - \mu_{bb})\mathbf{e}$ arises in the system on the Rabi frequency in addition to the Mollow triplet.[32] In our case intralevel motion is caused by interdot tunneling instead of parametrical oscillations. As a result, the difference $\mu_{aa} - \mu_{bb}$ in the spatially uniform case is replaced by $\xi_1 + \xi_2$.

IV. NANOANTENNAS BASED ON RABI WAVES

In this section, relying on the above-developed theory, we consider nanoantennas, based on Rabi waves. The utilization of Rabi waves in nanoantennas is possible both in optical and low-frequency ranges. The mechanism based on the Mollow triplet and leading to the excitation of polarization current (34) proves to be prospective in the optical range. The use of tunneling current (56) leads us to the low-frequency nanoantenna. Indeed, the tunneling ac-current spectrum covers the vicinity of the Rabi frequency Ω_R, and for realistic QDs and host materials this vicinity corresponds to the terahertz range.[15]

Radiative properties of antennas are strongly dependent on their configuration.[17] As an important step in nanoantenna design, the nanowire nanoantennas of different natures were proposed.[19,22,27] Their radiation patterns are similar to the radiation of an ideal electric dipole. The nanostructures with toroidal geometry have recently been discussed in Refs. 31, 33 and 34. Their radiation pattern is similar to the radiation of an ideal magnetic dipole. In our paper, as an illustration, we describe the performance of the terahertz-range loop Rabi-wave nanoantenna.

The statement of the problem is as follows. We consider a toroidal chain of QDs exposed to a whispering-gallery mode of the cylindrical microcavity. In this case the electromagnetic field satisfies the periodic boundary conditions, which leads to the discrete spectrum of the wave number k ($k = q/R_0$, where q is an integer, R_0 is the toroid's radius) and the Rabi frequency Ω_R is independent of the QD coordinate x. A ring-shaped QD chain placed coaxially nearby a ring-shaped metallic wire and interacting with the plasmon of the wire[34] is another example of the realization of such a type of antenna. We assume that the radius of the toroid R_0 is significantly larger than the chain period and is less than the wavelength $R_0 \gg a$, $kR_0 \ll 1$. The toroidal chain is considered as locally flat and described by Eq. (12) (the toroidal geometry is accounted for by the periodic boundary conditions). As a result, the current in the antenna is uniformly distributed around the ring with the radius R_0 and is defined by the expression (56). To calculate operator $\hat{f}_h$ we

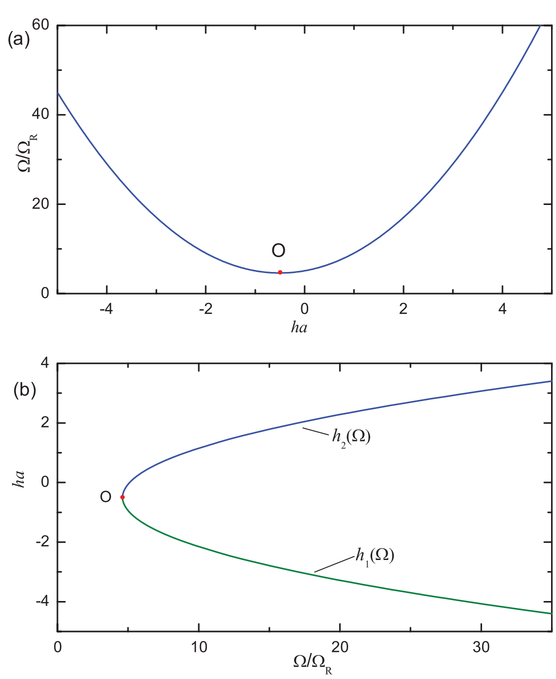

FIG. 3. (Color online) Dispersion characteristics of Rabi wave in the Ω-h plane. Case 1: $\Delta > -(h_0a)^2(\xi_2 - \xi_1)$.

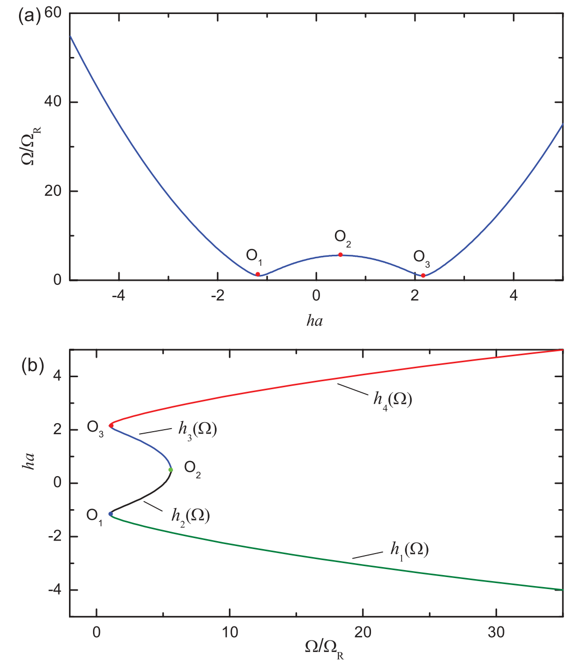

FIG. 4. (Color online) Dispersion characteristics of a Rabi wave in the Ω-h plane. Case 1: $\Delta < -(h_0a)^2(\xi_2 - \xi_1)$.

rewrite the expression (50) in the form of

$$\hat{f}_h = -\frac{R_0}{2\pi}\int_0^{2\pi} \hat{\rho}(R_0\cos\theta,0)e^{ihR_0\cos\theta}\sin\theta d\theta. \tag{58}$$

To apply general methods of antenna theory to the structure under consideration, the current (56) should be represented in the form of a frequency-domain Fourier integral. Let us consider the qualitative character of dispersion curves $\Omega = \Omega(h)$ [see Figs. 3(a) and 4(a)]. One can see from the figures that two different regimes are possible. The first one is depicted in Fig. 3(a) and corresponds to the case $\Delta > -(h_0a)^2(\xi_2 - \xi_1)$, $h_0 = -\kappa(\xi_2 + \xi_1)/2(\xi_2 - \xi_1)$ is the root of the equation $d\Delta_4/dh = 0$. The second case [Fig. 4(a)] corresponds to the inequality $\Delta < -(h_0a)^2(\xi_2 - \xi_1)$.

To pass in Eq. (56) to the frequency domain we need to invert the dependence $\Omega = \Omega(h)$ [i.e., to transform it into $h = h(\Omega)$]. Dependence $h(\Omega)$ is not single-valued because $\Omega(h)$ has local branching points. It means that the transition in Eq. (56) from the integration with respect to h to integration with respect to Ω should be performed separately for each branch of $h(\Omega)$. In the first case we have one local turning point O and, as a result, two branches $h_{1,2}(\Omega)$ [see Fig. 3(b)]. Taking this into account, after some transformations we obtain for the current (56) the expression as follows:

$$J^{(\Omega)}(t) = a^2 e\int_{-\infty}^{\infty} \tilde{P}(\Omega)e^{i\Omega t}d\Omega, \tag{59}$$

where

$$\tilde{P}(\Omega) = \begin{cases} P(\Omega), & \Omega > \Omega_{cr}, \\ 0, & |\Omega| < \Omega_{cr}, \\ P^*(\Omega), & \Omega < -\Omega_{cr}, \end{cases} \tag{60}$$

$$\Omega_{cr} = \sqrt{\Omega_R^2 + \left[\Delta + \frac{k^2a^2(\xi_2+\xi_1)^2}{4(\xi_2-\xi_1)}\right]^2}, \tag{61}$$

$$P(\Omega) = \mathrm{Tr}\big(\hat{f}_{h_1}\hat{\tau}_{1h_1}\big)\zeta_1(h_1)\frac{dh_1}{d\Omega} - \mathrm{Tr}\big(\hat{f}_{h_2}\hat{\tau}_{1h_2}\big)\zeta_1(h_2)\frac{dh_2}{d\Omega}, \tag{62}$$

where $\zeta_1(h)$ is given by Eq. (57).

Note that $P(\Omega)$ is not comprised of singular points at $\Omega = \pm\Omega_{cr}$, which we expect due to the condition $dh_{1,2}(\Omega_{cr})/d\Omega \to \infty$ because the divergent components in the first and second terms of Eq. (62) compensate each other.

For the second case, we have three local branching points $O_{1,2,3}$ and four branches $h_{1,2,3,4}(\Omega)$ [Fig. 4(b)]. For the tunneling current we obtain, after some manipulations, the expression as follows:

$$J_{\mathrm{tun}}^{(\Omega)}(t) = a^2 e\left[\int_{\Omega_R}^{\infty} P(\Omega)e^{i\Omega t}d\Omega + \int_{\Omega_R}^{\Omega_{cr}} Q(\Omega)e^{i\Omega t}d\Omega\right] + \mathrm{c.c.}, \tag{63}$$

where $Q(\Omega)$ can be obtained from Eq. (62) by changing $h_1 \to h_3(\Omega)$ and $h_2 \to h_4(\Omega)$.

Thus, we obtain a model of the nanoantenna excited by a spatially homogeneous nonharmonic ring current given by Eqs. (59) or (63). The radiation pattern of the antenna can be calculated from the current Fourier transformation with respect to Ω using standard methods of the antenna theory.[17] Let us introduce the spherical coordinate system with its origin in the center of the current ring. In the first case, the radiation field can be written in the form as follows:

$$E_\varphi = \frac{2\pi^2 R_0^3 ae}{c^3}\frac{\sin\Theta}{R}\int_{-\infty}^{\infty}\tilde{P}(\Omega)\Omega^2 e^{i\Omega(t-R/c)}d\Omega, \tag{64}$$

$$H_\Theta = -E_\varphi, \tag{65}$$

where Θ, φ are meridional and azimuthal coordinates, respectively, R is the distance from the coordinate origin. The field for the second case can be obtained in the same way with the current per unit cell given by Eq. (63) instead of Eq. (59). From Eq. (64) one can see that the radiation field of the nanoantenna is considered equivalent to the field of a magnetic dipole placed in the center of the current ring and oriented orthogonal to the ring plane. This equivalence is a consequence of the electrical smallness of the antenna $(\Omega R_0/c) \ll 1$ over the working range of the frequency spectrum.

It should be noted that the frequency spectrum of the antenna is bounded from the bottom by the finite value of the frequency $\Omega = \Omega_{cr}$ for the first case and $\Omega = \Omega_R$ for the second one. As a result, expression (64) can be rewritten in the form of

$$E_\varphi = \frac{2\pi^2 R_0^3 ae}{c^3}\frac{\sin\Theta}{R}[e^{i\Omega(t-R/c)}u(t-R/c) + \mathrm{c.c.}], \tag{66}$$

where $u(x) = \int_0^\infty P(\nu+\Omega_{cr})(\nu+\Omega_{cr})^2\exp(i\nu x)d\nu$. From Eq. (66) it follows that the radiated signal has the form of the amplitude-modulated oscillation with the carrier frequency Ω_{cr} and the amplitude modulation $u(x)$. The feature of the Rabi-wave antenna is the dependence of the carrier frequency on the electromagnetic field intensity. With the R_0 increase, the contribution of high-order magnetic multipoles becomes essential and the radiation pattern becomes more complicated:[17] $F(\Theta) \sim J_1(\Omega_{cr}R_0\sin\Theta/c)$, where J_1 is the Bessel function. Note that in this case $F(\Theta)$ depends on the carrier frequency and, consequently, on the electromagnetic field intensity. Such a peculiarity opens some useful opportunities for the electrical control of the radiation pattern of the Rabi-wave nanoantenna.

V. CONCLUSION

In the paper, a theory of mixed states has been developed for Rabi waves propagating in one-dimensional QD chains and interacting with classic light. The neighboring QDs are assumed to be tunneling coupled. The main results can be summarized in the following way.

To provide information transfer by Rabi waves their modulation is required. This modulation means that the spectrum finite width is achieved by mixing the chain quantum states. In the paper we have presented a formalism of the mixed states description based on the generalized Bloch equations.

The frequency spectrum of the current induced by the Rabi wave is comprised of the dc component, the low-frequency component, and the high-frequency component. For realistic QD-chain parameters the low-frequency component falls into the terahertz range nearby the Rabi frequency. The dc and low-frequency currents are due to electron tunneling along the chain.

The high-frequency current is in the optical range in the vicinity of the quantum transition frequency. The physical nature of this component is analogous to the Mollow triplet. However, as it is different from the standard Mollow triplet, in our case the satellites' spectrum is continuous due to the mixing.

The application of Rabi waves for the excitation of quantum nanoantennas with electrically controlled radiation pattern and frequency characteristics is proposed. It is shown that two types of Rabi-wave nanoantennas are possible. The first one, with the operation frequency in the visible range, is based on the high-frequency component of the current. The second one utilizes the low-frequency component and its operation frequency is in the terahertz range. As an example, the radiation pattern of the second type of antenna is formed as a QD ring interacting with the optical whispering-gallery mode has been considered. It has been demonstrated that the radiation properties of such an antenna are the same as the ideal magnetic dipole. The antenna frequency spectrum corresponds to the amplitude-modulated Rabi oscillations.

The theory of Rabi waves presented in the paper can be applied qualitatively to systems with inter-QD coupling of another physical nature. For example, for QDs placed nearby the metallic nanowire the coupling can be due to the plasmon excited in the wire.[35]

ACKNOWLEDGMENTS

The research was partially supported by the EU FP7 Projects No. FP7-230778 TerACaN and No. FP7-266529 BY-NanoERA, and BRFFR (Belarus) Project No. F10R-002. The work of G.Y.S. was partially carried out during the stay at the Institut für Festkörperphysik, TU Berlin, and supported by DFG.

*Jarchak@gmail.com

[1] I. I. Rabi, Phys. Rev. **51**, 652 (1937).
[2] L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Dover, New York, 1975).
[3] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, England, 2001).
[4] A. A. Svidzinsky, J.-T. Chang, and M. O. Scully, Phys. Rev. Lett. **100**, 160504 (2008).
[5] A. A. Svidzinsky, J.-T. Chang, and M. O. Scully, Phys. Rev. A **81**, 053821 (2010).
[6] S. Das, G. S. Agarwal, and M. O. Scully, Phys. Rev. Lett. **101**, 153601 (2008).
[7] M. O. Scully, Phys. Rev. Lett. **102**, 143601 (2009).
[8] T. Yu and J. H. Eberly, Phys. Rev. Lett. **93**, 140404 (2004).
[9] M. O. Scully, E. S. Fry, C. H. Raymond Ooi, and K. Wodkiewicz, Phys. Rev. Lett. **96**, 010501 (2006).
[10] A. Kavokin, J. J. Baumberg, G. Malpuech, and F. P. Laussy, *Microcavities* (Oxford University Press, New York, 2007).
[11] S. M. Dutra, *Cavity Quantum Electrodynamics: The Strange Theory of Light in a Box* (Wiley-Interscience, Hoboken, NJ, 2005).
[12] J. J. Hopfield, Phys. Rev. **112**, 1555 (1958).
[13] A. Quattropani, L. C. Andreani, and F. Bassani, Nuovo Cimento **7D**, 55 (1986).
[14] G. Ya. Slepyan, Y. D. Yerchak, S. A. Maksimenko, and A. Hoffmann, Phys. Lett. A **373**, 1374 (2009).
[15] G. Ya. Slepyan, Y. D. Yerchak, A. Hoffmann, and F. G. Bass, Phys. Rev. B **81**, 085115 (2010).
[16] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-Relativistic Theory* (Pergamon, Oxford, 1977).
[17] C. Balanis, *Antenna Theory: Analysis and Design* (Wiley-Interscience, Hoboken, NJ, 2005).
[18] P. Bhardwaj, B. Deutsch, and L. Novotny, Advances Optica Photonic **1**, 438 (2009).
[19] L. Novotny, Phys. Rev. Lett. **98**, 266802 (2007).
[20] L. Rogolete, F. Kaminski, M. Agio, and V. Sandoghdar, Opt. Lett. **32**, 1623 (2007).
[21] A. V. Akimov, A. Mukherjee, C. L. Yu, D. E. Chang, A. S. Zibrov, P. R. Hemmer, H. Park, and M. D. Lukin, Nature **450**, 402 (2007).
[22] A. Alu and N. Engheta, Nature Photon. **V2**, 307 (2008).
[23] R. Esteban, T. V. Teperik, and J. J. Greffet, Phys. Rev. Lett. **104**, 026802 (2010).
[24] L. Novotny and N. van Hulst, Nature Photon. **5**, 83 (2011).
[25] G. W. Hanson, IEEE Trans. Antennas Propag. **53**, 3426 (2005).
[26] P. J. Burke, S. Li, and Z. Yu, IEEE Trans. Nanotechnology **5**, 314 (2006).
[27] G. Ya. Slepyan, M. V. Shuba, S. A. Maksimenko, and A. Lakhtakia, Phys. Rev. B **73**, 195416 (2006).
[28] K. Kempa, J. Rybczynski, Z. Huang, K. Gregorczyk, A. Vidan, B. Kimball, J. Carlson, G. Benham, Y. Wang, A. Herczynski, and Z. F. Ren, Adv. Mater. **19**, 421 (2007).
[29] M. di Ventra, *Electrical Transport in Nanoscale Systems* (Cambridge University Press, Cambridge, England, 2008).
[30] L. P. Pitaevskii and E. M. Lifshitz, *Physical Kinetics* (Pergamon, Oxford, 1981).
[31] O. V. Kibis, Phys. Rev. Lett. **107**, 106802 (2011).
[32] O. V. Kibis, G. Ya. Slepyan, S. A. Maksimenko, and A. Hoffmann, Phys. Rev. Lett. **102**, 023601 (2009).
[33] T. V. Teperik and A. Degiron, Phys. Rev. B **83**, 245408 (2011).
[34] Z.-J. Yang, N.-C. Kim, J.-B. Li, M.-T. Cheng, S.-D. Liu, Z.-H. Hao, and Q.-Q. Wang, Opt. Express **18**, 4006 (2010).
[35] G. Y. Chen, N. Lambert, C. H. Chou, Y. N. Chen, and F. Nori, Phys. Rev. B **84**, 045310 (2011).