Physics Letters A 373 (2009) 1374–1378

Contents lists available at ScienceDirect

Physics Letters A

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Wave propagation of Rabi oscillations in one-dimensional quantum dot chain

G.Ya. Slepyan [a], Y.D. Yerchak [a,*], S.A. Maksimenko [a], A. Hoffmann [b]

[a] Institute for Nuclear Problems, Belarus State University, Bobruiskaya 11, 220050 Minsk, Belarus
[b] Institut für Festkörperphysik, Technische Universität Berlin, Hardenbergstrasse 36, 10623 Berlin, Germany

ARTICLE INFO

Article history:
Received 23 January 2009
Accepted 1 February 2009
Available online 11 February 2009
Communicated by V.M. Agranovich

PACS:
71.10.Li
71.36.+c
73.21.La

Keywords:
Rabi oscillations
Rabi waves
QD-chain
Electron tunneling

ABSTRACT

Interaction of traveling wave of classic light with 1D-chain of coupled quantum dots (QDs) in the strong coupling regime is theoretically considered. The effect of space propagation of Rabi oscillations in the form of traveling waves and wave packets is predicted introducing thus a new family of quasi-particles. Physical interpretation of the effect is given and principles of its experimental observation are discussed.

1. Introduction

Rabi oscillations are periodical transitions of a two-state quantum system between its stationary states in the presence of an oscillatory driving field, see e.g. [1]. Theoretically predicted by Rabi in 1937 [2], Rabi oscillations were firstly observed by Torrey in 1949 on nuclear spins in radio-frequency magnetic field [3]. Afterwards, this phenomenon was discovered in many other systems, such as electromagnetically driven atoms [4], semiconductor QDs [5], Josephson qubits [6], spin-qubits [7], and between ground and Rydberg atomic states [8]. Besides the fundamental interest, the effect of Rabi oscillations is promising for realization of binary logic and optical control in quantum informatics and quantum computing. Complication of physical systems where Rabi effect is observed imposes additional features on the ideal picture [1] of this effect. They are the time-domain modulation of the field-matter coupling constant [9,10], the phonon-induced dephasing [11], the local-field effect [12–14] and effect of broken inversion symmetry [15] – just to mention a few. New capabilities appear in systems of two coupled Rabi oscillators [16–22].

In spatially extensive samples comprising a great number of oscillators, the mechanism giving rise to Rabi oscillations induces also a set of nonstationary coherent optical effects, such as optical nutation, photon echo, self-induced transparency, etc. [23]. This is because the sample size exceeds significantly wavelength and propagation effects come into play. In low-dimensional systems propagation effects are also manifested but their character changes qualitatively. For example, the computational model of the coherent intersubband Rabi oscillations in a sample comprising 80 AlGaAs/GaAs quantum wells [24] predicts the population dynamics to be dependent on the quantum well position in the series. This result demonstrates strong radiative coupling between wells and, more generally, significant difference in the Rabi effect picture for single and multiple oscillators. In the present Letter we build for the first time a theoretical model of a distributed system of coupled Rabi oscillators and predict a new physical effect: propagation of Rabi oscillations in space in the form of traveling waves and wave packets.

2. Model and equation of motion

Consider an infinite periodical one-dimensional chain of identical QDs, containing an electron in the size-quantized conduction band. It is assumed, that in each pth QD there are at least two one-electron orbital states, ground $|b_p\rangle$ and excited $|a_p\rangle$, with transition frequency ω_0. Neighboring QDs are coupled via electron tunneling [22], i.e. the electron can change from state $|a_p\rangle$ to state $|a_{p\pm1}\rangle$ and from $|b_p\rangle$ to state $|b_{p\pm1}\rangle$. Transitions between ground state and exited state belonging to different QDs is neglected: $\langle a_{p\pm1}|b_{p\pm1}\rangle \approx 0$. Let the QD chain be exposed to a plane wave traveling along the

* Corresponding author.
E-mail address: jarchak@gmail.com (Y.D. Yerchak).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2009.02.016

chain, $E \sim \exp[-i(\omega t - kx)]$. The wavenumber satisfies the condition $ka \lesssim 1$ with a as the chain period, providing later on the continuous limit transition. In the one-particle basis, the Hamiltonian of the system is represented by $\hat{H} = \hat{H}_0 + \hat{H}_T + \Delta\hat{H}$, where the term

$$\hat{H}_0 = \frac{\hbar\omega_0}{2}\sum_n \hat{\sigma}_{zn} - \frac{\hbar\Omega_R}{2}\sum_n \left[\hat{\sigma}_n^+ e^{i(nka-\omega t)} + \text{H.c.}\right] \tag{1}$$

describes Rabi oscillations in non-interacting QDs, $\hat{\sigma}_{zn}$ and $\hat{\sigma}_n^+$ are the Pauli matrices for nth QD, Ω_R is the Rabi frequency [1]. The term $\Delta\hat{H}$ is for the local-field effects originated from the dipole–dipole electron–hole interaction [12–14]; in the mean-field approximation this term is given by

$$\Delta\hat{H} = \frac{4\pi}{\mathcal{V}}\mu(\underline{\tilde{N}}\mu)\sum_m \left(\hat{\sigma}_m^-\langle\hat{\sigma}_m^+\rangle + \hat{\sigma}_m^+\langle\hat{\sigma}_m^-\rangle\right), \tag{2}$$

where μ is the QD dipole moment, $\mathcal{V}$ is the volume of QD, $\underline{\tilde{N}}$ is the depolarization tensor (see Eq. (18) in Ref. [14]). The term $\hat{H}_T$ accounts for the QD-coupling:

$$\hat{H}_T = -\hbar\xi\sum_n\sum_{p=\pm1}\left(|a_n\rangle\langle a_{n+p}| + |b_n\rangle\langle b_{n+p}|\right), \tag{3}$$

where ξ is the matrix element for the electron tunneling between neighboring QDs.

Equation of motion has the form of one-particle Schrödinger equation $i\hbar\partial_t|\Psi\rangle = \hat{H}|\Psi\rangle$ with the wave function written as $|\Psi(t)\rangle = \sum_p (A_p(t)|a_p\rangle + B_p(t)|b_p\rangle)$. Taking into account (1) and (3), we reduce the Schrödinger equation to a system of difference-differential equations, which directly couples A_p with $B_p, A_{p\pm1}$ and B_p with $A_p, B_{p\pm1}$. Carrying out then the continuous limit transition for the variable $A_p(t)$ by $A_p(t) \to A(x,t)$, $A_{p+1}(t) + A_{p-1}(t) - 2A_p(t) \to a^2\partial_x^2 A(x,t)$ and in the same manner for the variable $B_p(t)$, in the rotating-wave approximation [1] we arrive at the system of partial differential equations as follows:

$$\partial_t A = -\frac{i}{2}(\omega_0 - 4\xi)A + \frac{i\Omega_R}{2}Be^{i(kx-\omega t)} + i\xi a^2\partial_x^2 A - i\Delta\omega|B|^2 A, \tag{4}$$

$$\partial_t B = \frac{i}{2}(\omega_0 + 4\xi)B + \frac{i\Omega_R}{2}Ae^{-i(kx-\omega t)} + i\xi a^2\partial_x^2 B - i\Delta\omega|A|^2 B, \tag{5}$$

where $\Delta\omega = 4\pi\boldsymbol{\mu}(\underline{\tilde{N}}\boldsymbol{\mu})/\hbar\mathcal{V}$ is the local-field induced depolarization shift [14]. As it is seen from Eqs. (4)–(5), two concurrent mechanisms manifest themselves in the light – QD chain coupling: the local-field induced nonlinearity and the dispersion spreading due to the tunneling. In the given Letter we restrict consideration to linear regime of the carrier motion and omit the terms $O(\Delta\omega)$ in (4)–(5). The conditions allowing such a model are formulated below.

The Hamiltonian $\hat{H}$ does not account for the dephasing and relaxation processes. Generally, to allow for them one should solve the many-particle problem. But we can take decay into consideration phenomenologically (see Problem 5.2 in [1]), substituting $\omega_0 \to \omega_0 - i\lambda$ into Eq. (4) and $\omega_0 \to \omega_0 + i\lambda$ into Eq. (5), where λ is damping constant.

3. Traveling Rabi waves

Assuming $\Delta\omega = 0$ and introducing dissipation, let us consider elementary solution of the system (4)–(5) in the form of traveling wave: $A \sim e^{i(h+k/2)x}e^{-i(\nu+\omega/2)t}e^{-\lambda t/2}$, $B \sim e^{i(h-k/2)x}e^{-i(\nu-\omega/2)t}e^{-\lambda t/2}$, where h is a given wave number and ν is the eigenfrequency to be found. Solving the characteristic equation of the system (4)–(5) with respect to ν we determine the system eigenfrequencies as

$$\nu_{1,2} = \xi a^2h^2 - \Phi \pm \left[\left(\frac{\Delta}{2} + Vh\right)^2 + \frac{\Omega_R^2}{4}\right]^{1/2}, \tag{6}$$

where $\Delta = \omega_0 - \omega$ is the frequency detuning, $V = \xi ka^2$ and $\Phi = 2\xi(1 - k^2a^2/8)$.

As analysis carried out in [14] has shown, two essentially different regime of Rabi oscillations are possible in QDs. In first regime the frequency of Rabi oscillations exceeds the depolarization shift. In this case Rabi oscillations are strongly pronounced: the inversion oscillates in the range $[-1, 1]$ and the influence of local-field effects is negligibly small. For the problem under consideration this regime corresponds to the condition

$$\nu_{1,2}(h) \gtrsim \Delta\omega, \tag{7}$$

which allows neglecting terms $O(\Delta\omega)$ in (4)–(5).

Two solutions (6) correspond to two eigenmodes, given, respectively, by

$$A_1(x,t) = -\frac{C_1\Omega_R e^{i(h+k/2)x}e^{-i(\nu_1+\omega/2)t}e^{-\lambda t/2}}{2(\nu_1 + \Phi - \Delta/2 - Vh - \xi a^2h^2)},$$
$$B_1(x,t) = C_1 e^{i(h-k/2)x}e^{-i(\nu_1-\omega/2)t}e^{-\lambda t/2}, \tag{8}$$

and

$$A_2(x,t) = C_2 e^{i(h+k/2)x}e^{-i(\nu_2+\omega/2)t}e^{-\lambda t/2},$$
$$B_2(x,t) = -\frac{C_2\Omega_R e^{i(h-k/2)x}e^{-i(\nu_2-\omega/2)t}e^{-\lambda t/2}}{2(\nu_2 + \Phi + \Delta/2 + Vh - \xi a^2h^2)}, \tag{9}$$

where $C_{1,2}$ are normalizing constants. Either of these modes is a *superposition* of ground and excited states, whose partial amplitudes *oscillate both in time and space*. Binding of ground and excited states is caused by interaction of light with QD chain and vanishes in the limit of $\Omega_R \to 0$. In that case, (4)–(5) describe QD-chain excitons in equilibrium and inverse states, respectively. Retaining in the expansion terms linear in $|\Omega_R|$ and simultaneously substituting $\Delta \to \Delta - i0$ we arrive at the intermediate case of excitons weakly coupled with electromagnetic field.

Space oscillations of the partial amplitudes are due to QD-coupling and vanishes in the limit of $\xi \to 0$. Thus, each of these modes can be interpreted as a Rabi wave with the frequency determined by Eq. (6). In general case these waves are excited simultaneously, while any of them can be excited separately by a proper choice of initial conditions. Similarly to other coherent excitations in condensed matter, Rabi waves (8)–(9) introduce a new family of quasi-particles (we can call them as *rabitons*), which the standard secondary quantization technique is applicable to. Similarly to the effect of the self-induced transparency, the Rabi wave propagation can be interpreted as the motion of a precessing pseudo dipole [23]. However, the coherence mechanisms in these two cases are different: In the Rabi wave the coherence is settled by the dispersion relation (6) while in the case of the self-induced transparency it has solitonic nature.

Typical dispersion characteristics of Rabi waves are depicted in Fig. 1. It should be noted that at given amplitude and frequency

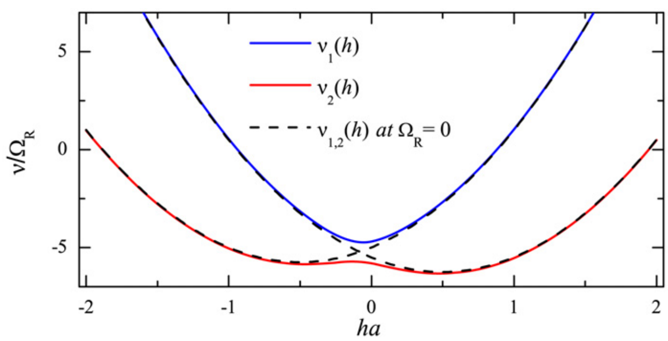

Fig. 1. Typical dispersion law for Rabi waves. $\xi = 3\Omega_R$, $\Delta = 0.5\Omega_R$, $ka = 1$.

of external field their frequencies have continuous spectra (wave number h varies continuously). Dispersion dependences depicted in the figure are asymmetric: $\nu_{1,2}(h) \neq \nu_{1,2}(-h)$. Physically, this is due to the presence of preferential direction, which is determined by the direction of light propagation along the chain. The QD coupling leads to the inequality $\nu_1 \neq -\nu_2$. That is why, unlike to single QD, in QD chains the inversion oscillates anharmonically. These oscillations can be represented as amplitude-modulated harmonic oscillations with the frequency $(\nu_1 - \nu_2)/2$, while the modulation frequency is given by $(\nu_1 + \nu_2)/2$. In traveling Rabi wave, the inversion is constant in space. Because $\nu_{1,2}(h)$ are real at any real h, the system is stable [25].

Note that at certain h the condition $\nu_{1,2}(h) = 0$ is fulfilled. In the vicinity of these points the condition (7) is changed to opposite. In accordance with [14] Rabi oscillation in this case are weakly marked (the amplitude of inversion oscillations is small). Consequently, dynamics of the system is determined to the large degree by the local-field effect. As a result, neglecting $\Delta\omega$ in (4)–(5) is unwarranted and the problem needs a special consideration.

Assuming the frequency ν to be a given parameter and solving Eq. (6) with respect to wave number, we obtain for $k = 0$:

$$h_{1,2} = \pm\left[\frac{1}{a^2\xi}\left(\nu + 2\xi \pm \frac{1}{2}\sqrt{\Omega_R^2 + \Delta^2}\right)\right]^{1/2}, \tag{10}$$

where external signs $\pm$ correspond to two directions of propagation while signs $\pm$ before internal radical correspond to two types of Rabi waves indexed by 1 and 2. It is seen that for real ν, in a certain frequency range h_1 is complex and h_2 is real. It corresponds to non-transmission of the first Rabi wave [25]. The frequency range in which both of $h_{1,2}$ are complex also exists. This case corresponds to complete non-transmission of the Rabi waves with the given frequency.

Note that the eigenmodes (8) and (9) each comprise traveling waves with *different* wave numbers $h \pm k/2$. Physically, this means that the Rabi wave propagates in an effective periodically inhomogeneous medium formed by spatially oscillating (with period $2\pi/k$) electric field. Therefore, the diffraction is developed in the system. In the limit $k \to 0$ the medium turns homogeneous and the diffraction effect vanishes. Reflection of Rabi waves and their mutual transformations at inhomogeneities becomes possible. In that way one obtain a unique ability to control the processes of the reflection and dispersion of Rabi waves by varying the light spatial distribution.

4. Rabi wave packets

Let us analyze the general solution of the system (4)–(5). To find it, we first introduce the new variables $u_+(x,t) = A(x,t) \times \exp[i(\omega t - kx - 2\Phi t - i\lambda t)/2]$, $u_-(x,t) = B(x,t)\exp[-i(\omega t - kx + 2\Phi t - i\lambda t)/2]$. For these variables, Eqs. (4)–(5) are reduced to the form as follows:

$$\partial_t u_\pm \pm \frac{i\Delta}{2} u_\pm \pm V\partial_x u_\pm - i\xi a^2 \partial_x^2 u_\pm - \frac{i\Omega_R}{2} u_\mp = 0. \tag{11}$$

This system can be solved exactly by using the Fourier transform with respect to x. Finally, we arrive at

$$u_\pm(x,t) = \int_{-\infty}^{\infty} \left[v_\pm(h)\varphi_h^\mp(t) + v_\mp(h)\psi_h(t)\right] e^{ih(x - \xi a^2 h t)}\, dh, \tag{12}$$

where $\psi_h(t) = i(\Omega_R/\Omega_h)\sin\tau$, $\varphi_h^\pm(t) = \cos\tau \pm i(\Delta_h/\Omega_h)\sin\tau$, $\tau = \Omega_h t/2$, $\Omega_h = \sqrt{\Omega_R^2 + \Delta_h^2}$, and $\Delta_h = \Delta + 2Vh$. The functions $v_\pm(h)$ are determined by the initial condition

$$v_\pm(h) = \frac{1}{2\pi}\int_{-\infty}^{\infty} u_\pm(x,0) e^{-ihx}\, dx. \tag{13}$$

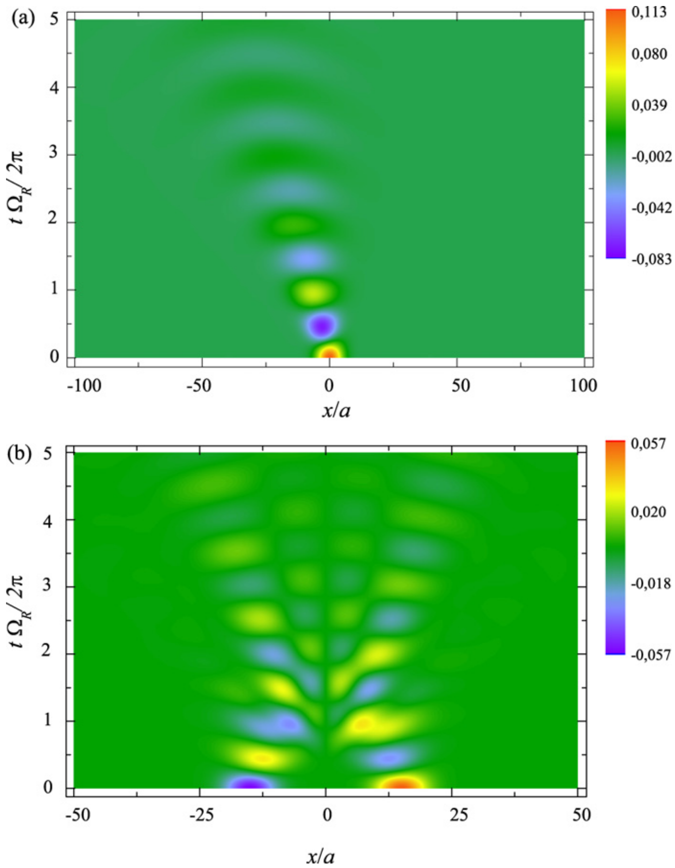

Fig. 2. Space–time distribution of the inversion in the QD chain. (a) A single Gaussian wavepacket $A(x,0) = (1/\sqrt[4]{\pi\sigma^2})\exp(-x^2/2\sigma^2)$, $B(x,0) = 0$, $\Delta = Vk$. (b) Two counterpropagating identical Gaussian wavepackets: $A(x,0) = \exp[-(x - 3\sigma)^2/2\sigma^2]/\sqrt[4]{4\pi\sigma^2}$, $B(x,0) = \exp[-(x + 3\sigma)^2/2\sigma^2]/\sqrt[4]{4\pi\sigma^2}$, $\Delta = 0$. In both cases $ka = 0.33$, $\sigma = 5a$, $\xi = 3\Omega_R$, $2\pi\lambda/\Omega_R = 0.1$.

A typical space–time distribution of the spatial density of the inversion (the inversion per single QD) $w(x,t) = a[|A(x,t)|^2 - |B(x,t)|^2] = a[|u_+(x,t)|^2 - |u_-(x,t)|^2]e^{-\lambda t}$ is shown in Fig. 2(a). As follows from Eqs. (12) the depicted space–time dynamics corresponds to the evolution of a Rabi wavepacket defined as a superposition of Rabi waves with continuous spectrum of Ω_h. Physical interpretation of the picture predicted to observe in the QD chain can be given on the base of the collapse-revivals concept [1]. Distinctive feature of the case considered is that the distribution of collapses and revivals is permanently varied in space and time. Although variation of the inversion density, depicted in Fig. 2, in arbitrary point of the space occupied by the Rabi wavepacket is not too large, an integral characteristics presented in Fig. 3 – the "integral" inversion $\int_{-\infty}^{\infty} w(x,t)\,dx$ – of initially unpolarized QD-chain oscillates between -1 and 1, thus indicating presence of strong light-QD coupling.

Note that oscillations of the integral inversion at $V \neq 0$ damp with time even at $\lambda = 0$ (see Fig. 3), whereas such a damping is absent at $V = 0$ and integral inversion oscillates harmonically in the range from -1 to 1 (dotted curve in Fig. 3). Such a behaviour indicates appearance of a specific mechanism of collective dephasing. Physically, this is because the effective detuning Δ_h and therefore the carrier frequency Ω_h of Rabi oscillations constituting the wavepacket depends on h (Doppler shift). As dif-

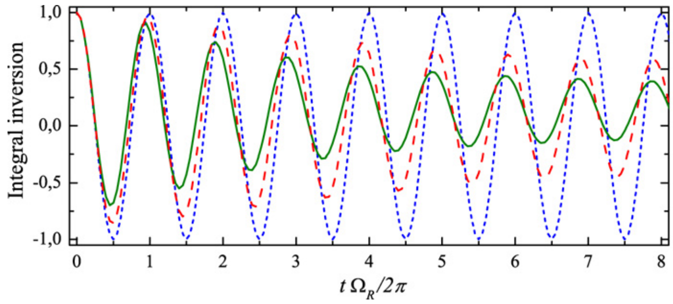

Fig. 3. Temporal dependence of the integral inversion at the input parameters as follows: $\Delta = 0$, $k = 0$ (dotted line); $\Delta = 0$, $ka = 0.33$ (solid line); $\Delta = Vk$, $ka = 0.33$ (dashed line). In all cases, $\xi = 3\Omega_R$, $\sigma = 5a$ and $\lambda = 0$.

ferent from that, the condition $V = 0$ keeps the h-dependence only in the amplitude modulation frequency $(\nu_1 + \nu_2)/2$ and thus does not result in dephasing. In the weak coupling limit the indicated dephasing mechanism is analogous to the Landau damping in plasma.

The dependence of the frequency of Rabi oscillations Ω_h on V shows that the value $\Delta = 0$ is not optimal for the effect observation. Optimization of Δ allows increasing the intensity of Rabi wave and the dephasing time, see dashed curve in Fig. 3. In the frequency domain, fine tuning of the system to the resonance is achieved by the variation of V (changing the angle of incidence of light).

Interaction of two counterpropagating identical Gaussian Rabi wavepackets elastically colliding at $x = 0$ is shown in Fig. 2(b). Although the inversion oscillates in time and moves in space, integral inversion of initially saturated QD chain ($\int_{-\infty}^{\infty} w(x,0)\,dx = 0$) does not experience oscillations: this quantity equals zero for all $t \geqslant 0$ and arbitrary values of Ω_R.

For wavepackets, the condition $\nu_{1,2}(h) \gtrsim \Delta\omega$ of the local-field effects to be negligible must be fulfilled for any h essential in the spatial spectra of $u_{\pm}(x,0)$. At a given $\hbar\Delta\omega$, the validity of this assumption can be controlled by means of the dispersion curves presented in Fig. 1. For example, for 6 nm radius InGaAs QDs $\hbar\Delta\omega \sim 0.1$ meV [14], and at $\hbar\Omega_R \sim 1$ meV the local-field effects are negligible in a wide range of parameters ξ, h.

5. On experimental observability of Rabi waves

Theoretical analysis carried out has shown that the optimal excitation of Rabi waves require the coupling factors of both neighboring QDs and single QD with field to be comparable by magnitude: $\xi \sim \Omega_R$. Consequently, based on the above stated condition $\hbar\Omega_R \sim 1$ meV we estimate the interdot coupling constant by $\xi \sim 1$ meV,[1] which is reachable value [22] for typical interdot distances 4–20 nm.

When modeling the Rabi wavepacket propagation, we suppose the decay time approximately ten times bigger then the Rabi oscillations period. For $\hbar\Omega_R \sim 1$ meV it amounts ~ 50 ps, that is quite realistic value [26,27].

Experimentally, the preparation of an initial wavepacket in the QD chain can be realized by means of the resonant excitation of a certain QD through a 50–100 nm pinhole in a metallic mask. The pump-probe technique using the same pinhole can be applied for the detection by fixing nonstationary scattering pattern due to the wavepacket motion. Of course, highly ordered chains of uniform QDs are required to exclude nonhomogeneous broadening, which may hide the effect. Impressive progress in growing of perfect nanostructured successions achieved in last years (e.g., see [28]) is very promising for that aim.

6. Conclusion

In this Letter, we have predicted the existence of Rabi waves – wave propagation of the inversion in spatially extensive systems of coupled oscillators. The system has been exemplified by an 1D-chain of coupled QDs exposed to an intensive traveling light wave. Spatial propagation of Rabi oscillations in the form of traveling waves and wave packets is shown to occur in the chain. The propagation is predicted to be accompanied by the damping of Rabi wave in time manifesting new mechanism of collective dephasing, which is an analog of Landau damping of exciton-polaritons extended to the strong light-matter coupling.

The model of Rabi waves presented by Eqs. (11) can be applied to different systems of another physical nature, strongly coupled to electromagnetic field, such as semiconductor charge qubit chains [29]. Quantum electrical circuits can also be mentioned [6,30], if one proceed from two coupled Josephson qubits in microstrip resonator [6] to a distributed structure of such elements imposed to interqubit interaction. Note that in this structure the Rabi wave frequencies go down to microwaves.

Acknowledgements

Authors acknowledge a support from the INTAS (grant 05-1000008-7801). The work of S.A.M. was partially carried out during the stay at the Institut für Festkörperphysik, TU Berlin, and supported by the Deutsche Forschungsgemeinschaft (DFG). Authors are grateful to Dr. J. Haverkort for stimulative discussions.

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