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Rabi oscillations a quantum dot exposed to quantum light

A. Magyarov [a,*], G.Ya. Slepyan [a], S.A. Maksimenko [a], A. Hoffmann [b]

[a] *Institute for Nuclear Problems, Belarus State University, Bobruiskaya 11, 220050 Minsk, Belarus*
[b] *Institut Fuer Festkourperphusik, Technische Universitaet Berlin, Hardenbergstr. 36, 10623 Berlin, Germany*

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Abstract

The influence of the local field on the excitonic Rabi oscillations in an isolated quantum dot driven by the coherent state of light has been theoretically investigated. Local field is predicted to entail the appearance of two oscillatory regimes in the Rabi effect separated by the bifurcation. In the first regime Rabi oscillations are periodic and do not reveal collapse–revivals phenomenon, while in the second one collapse and revivals appear, showing significant difference as compared to those predicted by the standard Jaynes-Cummings model.

Keywords: Quantum dot; Local field; Rabi oscillations; Collapse–revivals phenomenon

Optical and electronic properties of quantum dot (QD) structures are currently in the focus of the research activity owing to their promising potentiality in different areas of present-day science and technology, such as in semiconductor device physics [1] or in quantum information storage and processing [2]. During the last decade attempts have been made to reveal the correspondence and to establish principle differences between atomic two-level systems and excitons in QDs. Many analogies have been investigated, such as the excitonic optical Stark effect, photon echoes in four-wave mixing experiments and excitonic Rabi oscillations (RO) in QDs. Excitonic RO in QDs are very promising: observed experimentally in number of papers, e.g. [3], they correspond to the one-qubit rotation that is the step towards the QD application in quantum information processing [3].

There are generally two types of the RO in an atomic two-level system. The first one appears in the strong coupling regime between the system and a classical field. As a result the population inversion periodically oscillates at the Rabi frequency [4]. The second type of RO is realized when a two-level system is imposed to quantum light. In this case the collapse–revivals phenomenon manifests itself in the time evolution of the population inversion [4]. A peculiarity that distinguishes the QD exciton from the atomic two-level system is the pronounced role of the local field (LF). LF effects are related to a difference between acting and mean fields in statistically large ensembles of particles like dense gases and artificial composite materials. Within the framework of macroscopic electrodynamics, LF in isolated QD is due to the external field screening by charges induced on the QD boundary [5]. The physical origin of LF is the exchange interaction between electron and hole forming the QD exciton (see Ref. [6] for details). In a QD interacting with a classical electromagnetic excitation in the strong-coupling regime, the LF leads to the bifurcation and anharmonism in the RO and the nonlinear dependence of the RO period on the incident field strength [7]. The latter result has been experimentally observed in isolated quantum islands confined in a single quantum well [8].

The LF influence on the RO in an isolated QD imposed to the quantized field has not been investigated so far. This problem is analyzed in the present paper. As an example of the quantum excitation, coherent field is considered. We demonstrate that the light–QD interaction is characterized by two coupling parameters: the standard Rabi frequency and the new one, the LF coupling constant. As a result, two types of oscillatory regimes are inherent to the Rabi effect, showing significant difference as compared to the standard collapse–revivals phenomenon.

Let an arbitrary-shaped QD be exposed to quantum light. In the quantum electrodynamics, the electron–hole exchange interaction is transferred by virtual vacuum photons [5,9,10]. Thus, in the dipole approximation, the system is described by

* Corresponding author.
E-mail address: andrei.magyarov@gmai.com (A. Magyarov).

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the Hamiltonian as follows: $H = H_0 + H_{ph} + H_{vac} + H_{I0} + H_{Iv}$, where $H_{0,\ ph,\ vac}$ are the Hamiltonians of free charge carriers, incident photons, and vacuum virtual photons, respectively. The Hamiltonians $H_{I0,Iv} = (1/2)\int_V (\hat{P}\hat{E}_{0,v} + \hat{E}_{0,v}\hat{P})\mathrm{d}^3r$ describe the interaction of electron–hole pairs with the incident quantum field $\hat{E}_0$ and the vacuum field $\hat{E}_v$, respectively. Here V is the QD volume and $\hat{P}(r, t)$ is the time-domain polarization operator. Note that the exchange by vacuum photons occurs between all allowed dipole transitions, so that on this stage the problem is stated as the quantum-mechanical many-body problem. For simplification of the problem we (i) exclude the vacuum photons' operators in the Hamiltonian H and carry out convolution of wavefunctions over corresponding variables; (ii) apply the Hartree-Fock-Bogoliubov approximation introducing thus a two-level system [11]. On this step, nonresonant transitions thus are approximately accounted through a real-valued and frequency-independent background dielectric function ε_h. Assuming ε_h to be equal to dielectric function of surrounding medium, we put $\varepsilon_h = 1$ without loss of generality [5]; (iii) admit the quasi-static approximation for the LF inside QD. Leaving details for a full-length publication, we state the result of the steps prescribed: the initial rigorous Hamiltonian is transformed into the two-body Hamiltonian $H_{eff} = H_{JC} + \Delta H$, where $H_{JC} = H_0 + H_{I0} + H_{ph}$ is the Jaynes-Cummings Hamiltonian [5] and ΔH is the correction accounting for the LF in the QD [5]:

$$\Delta H = (4\pi/V)(\mu \cdot \underline{N}\mu)\left(\hat{b}\langle\hat{b}^+\rangle + \hat{b}^+\langle\hat{b}\rangle\right) \tag{1}$$

Here, $\hat{b}^+$ and $\hat{b}$ are the creation and annihilation operators for the electron–hole pair and μ is its dipole moment, angular brackets $\langle\ldots\rangle$ stand for the operator's mean value. In the strong confinement regime, the depolarization tensor $\underline{N}$ is completely determined by the QD shape; see Ref. [5] for explicit expressions. The transition $H \rightarrow H_{eff}$ means the replacement of the carrier–vacuum interaction by the direct electron–hole interaction described by the term ΔH. Assuming the QD interaction with the single mode light in the rotating wave approximation, the nonlinear equations of motion

$$i\frac{\mathrm{d}A_n}{\mathrm{d}t} = \Omega_n B_{n+1} e^{i\delta t} + \Delta\omega B_n \sum_{m=0}^{\infty} A_m B_m^*,$$
$$i\frac{\mathrm{d}B_{n+1}}{\mathrm{d}t} = \Omega_n^* A_n e^{-i\delta t} + \Delta\omega A_{n+1} \sum_{m=0}^{\infty} A_m^* B_m \tag{2}$$

can be obtained from the Hamiltonian H_{eff}. In these equations A_n and B_n are functions to be found, index n denotes the number of photons in the field state; $\Omega_n = g\sqrt{n+1}$ is the quantum Rabi frequency; g is the frequency dimension, light–QD coupling constant; $\delta = \omega_0 - \omega$ is the frequency detuning, where $\omega_0(\omega)$ denotes the excitonic transition (carrier field) frequency. The parameter $\Delta\omega = 4\pi\underline{N}/\hbar V)|\mu|^2$ is the LF coupling constant (depolarization shift) [5]. It should be noted that Eq. (2) was derived under assumption of the strong confinement for the QD exciton. Thus, the analysis is restricted to the so-called small QDs, i.e., whose linear dimension is less than exciton Bohr radius. The case of large QDs (whose linear dimension is more than Bohr radius), without taking LF into account, has been considered in Ref. [12].

Let the ground-state QD be exposed to the elementary coherent state of light $|s\rangle = \sum_{n=0}^{\infty} F_n(0)|n\rangle$, where $F_n(0) = \exp[-\langle n(0)\rangle/2]\langle n(0)\rangle^{n/2}/\sqrt{n!}$ and $\langle n(0)\rangle$ denotes the mean number of photon in the initial time. Then, the initial conditions for Eq. (2) are: $A_n(0) = 0$ and $B_n(0) = F_n(0)$. We characterize the depolarization shift by the parameter $\xi = \Omega_{\langle n\rangle}/\Delta\omega$, where $\Omega_{\langle n\rangle} = 2g\sqrt{\langle n(0)\rangle + 1}$ is the average Rabi frequency [4]. Such definition is analogous to that introduced in our previous papers [7]. Consider a lossless system at the exact synchronism regime $\delta = 0$. In Fig. 1 results of the numerical integration of Eq. (2) for the QD population inversion defined as $w(t) = \sum_n (|A_n(t)|^2 - |B_n(t)|^2)$ for $\langle n(0)\rangle = 9.0$ and several values of ξ are represented. Calculations show the appearance of two completely different oscillatory regimes in the RO. The first one manifests itself at $\xi < 0.5$ and is characterized by the periodic oscillations of the inversion within the range $-1 \le w(t) \le 0$ (see Fig. 1a, b). On the contrary, in the second regime, which occurs at $\xi > 0.5$, the inversion oscillates in the range $-1 \le w(t) \le 1$ (Fig. 1c–f). These two oscillatory regimes are separated by the bifurcation at $\xi = \xi_b \cong 0.5$ (compare Fig. 1b and c). At large ξ, $\xi \ge 18.0$ (see, Fig. 1f), the inversion behavior is identical with that following from the standard JC model of the two-level system, where the LF is eliminated [4]. The appearance of two oscillatory regimes in the Rabi effect separated by the bifurcation at $\xi = 0.5$ has been predicted in [7] for an isolated QD driven by CW field. Note that in the vicinity of the bifurcation, the RO behavior is chaotic (see Fig. 1c, d) and drastically different from the collapse–revivals phenomenon predicted by the JC model (compare with Fig. 1f). The chaotic behavior will manifest itself as an appearance of the continuous background in the QD resonant fluorescence spectrum, which can be observed experimentally. Also, the LF may induce an additional revival in the time-evolution of the inversion (see Fig. 1e). It should be noted that at $\xi < 0.5$, the inversion behavior is identical to the classical driven field considered in Ref. [7] (see Fig.1a, b).

For the spherical GaAs QD with the radius $R \approx 3$ nm, the estimate $\hbar\Delta\omega \approx 1$ meV is obtained in Ref. [5]. This corresponds to the value of $\hbar\Omega_{\langle n\rangle} \approx 0.5$ meV for the case $\xi_b \cong 0.5$. Such values of Rabi frequencies are reachable in the pump-probe experiments [13].

The physical origin for the oscillatory regime at large ξ, $\xi > 18.0$, is the QD exciton dressing by incident photons, which is conventional for RO. To elucidate the origin for the oscillations at $\xi < 0.5$ let us consider the QD macroscopic polarization $\langle P\rangle = \mu V\langle b^+\rangle + \text{c.c.}$ in the limit when light–QD interaction is absent ($g = 0$). Then, after some manipulations with Eq. (2) the expression as follows

$$\langle P\rangle = \frac{1}{V}\mu a_0 b_0^* e^{-i(\omega_0 - \delta')t} + \text{c.c.}, \tag{3}$$

can be obtained. In this relation a_0 and b_0 are arbitrary constants satisfying $|a_0|^2 + |b_0|^2 = 1$. Parameter $\delta' = \Delta\omega w$ plays the role of the self-induced detuning, which depends on the inversion, i.e.,

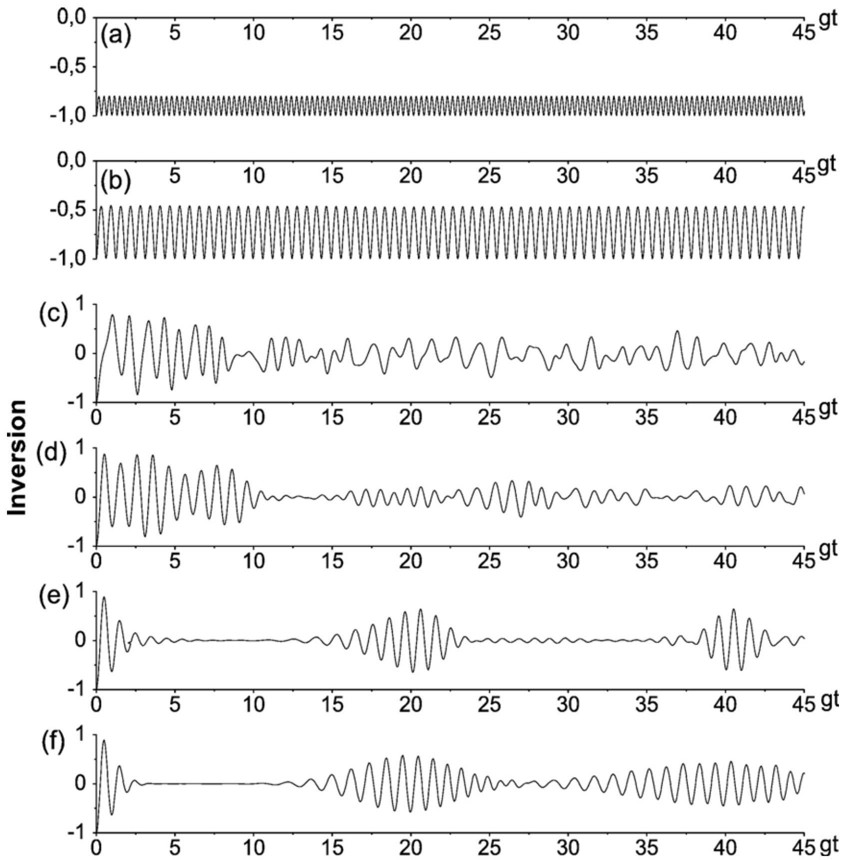

Fig. 1. Population inversion as a function of time: (a) $\xi=0.2$, (b) $\xi=0.49$, (c) $\xi=0.53$, (d) $\xi=1.2$, (e) $\xi=3.5$, (f) $\xi=18.0$.

on the state occupied by the exciton and on the depolarization shift. This, in turn, also influences the RO in the system: large values of δ' lead to the appearance of small amplitude oscillations (see Fig. 1a, b), as observed experimentally in Ref. [8]. Note that since the inversion may take values from the range $-1\leq w\leq 1$, from Eq. (3) it follows that the polarization oscillates at frequency ω_{pol} which satisfy the relation: $|\omega_{pol}-\omega_0|\leq\Delta\omega$. Consequently, when $g\neq 0$ the RO in the system are represented by a nonlinear superposition of two types of oscillations, (i) the polarization oscillations due to accounting for the LF and (ii) the standard RO.

The important characteristic inherent to RO is the change in the photonic state distribution $p_n(t)$ of incident light during its interaction with the quantum oscillator [4]. This is described by the second-order time-zero correlation function: $g^2(t)=\sum_n n(n-1)p_n(t)/[\sum_n np_n(t)]^2$ where $p_n(t)=\sum_n(|A_n(t)|^2+|B_n(t)|^2)$ [10], which can be measured in the single QD spectroscopy experiments (see, e.g. Ref. [14]). The initial distribution in coherent state of light is Poisson: $p_n(0)=|F_n(0)|^2$ that corresponds to $g^2(0)=1$ [4]. Results of calculations of $g^2(t)$ are shown in Fig. 2 for different values of ξ. At large ξ, $\xi>18.0$, and at $\xi<0.5$ $g^2(t)$ oscillates in the vicinity of unity. This indicates that the distribution $p_n(t)$ is varying in time, but remains Poisson. The behavior of $g^2(t)$ for $\xi>18.0$ is in agreement with the standard JC model [4] (see Fig. 2). Situation is changed at the vicinity of the bifurcation. Thus, at $\xi=1.2$, the function $g^2(t)$ demonstrates growth as the time is increased. This signifies that the photonic state distribution becomes super-Poisson that corresponds to the chaotic behavior in the RO of the inversion (compare with Fig. 1c, d).

In summary, the local field influence on the Rabi oscillations in the QD interacting with the coherent state of light has been analyzed. The QD population inversion demonstrates the appearance of two oscillatory regimes with drastically different characteristics, separated by the bifurcation. In the first regime oscillations are periodic and do not reach the inverted state. The collapse–revivals phenomenon is absent. In the second oscillatory regime, the collapse and revivals appeared in the time evolution of the inversion, however revivals are deformed

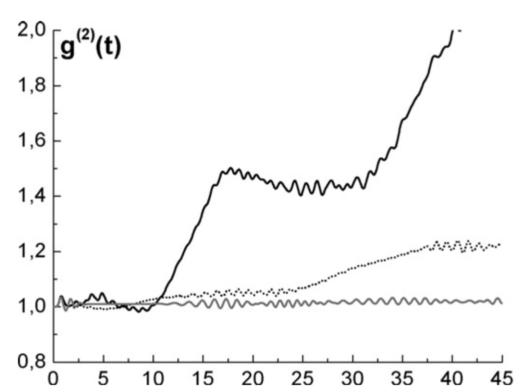

Fig. 2. Second order time-zero correlation function $g^2(t)$ as a function of time: $\xi=1.2$ (black line), $\xi=3.5$(dashed line), $\xi=18.0$ (grey line).

and significantly different from those predicted in the standard JC model of the two-level system. In the vicinity of the bifurcation, the noticeable modification of the second order time-zero correlation function has been obtained.

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