phys. stat. sol. (c) **2**, No. 2, 850–853 (2005) / **DOI** 10.1002/pssc.200460345

Strong light-matter coupling in a quantum dot: local field effects

G. Ya. Slepyan[1], **A. V. Magyarov**[1], **S. A. Maksimenko**[*, 1], **A. Hoffmann**[2], and **D. Bimberg**[2]

[1] Institute for Nuclear Problems, Belarus State University, Bobruiskaya 11, 220050 Minsk, Belarus
[2] Institut für Festkörperphysik, Technische Universität Berlin, Hardenbergstr. 36, 10623 Berlin, Germany

Received 29 June 2004, revised 12 July 2004, accepted 14 January 2005
Published online 15 February 2005

PACS 42.50.Ct,73.21.–b,78.67.Hc

Manifestation of local fields in excitonic Rabi oscillations in an isolated arbitrary shaped quantum dot (QD) has been theoretically investigated. Interaction of QD with harmonic electromagnetic field as well as with ultrashort optical pulse has been considered. Bifurcation and anharmonism in the Rabi oscillations in a QD are predicted. Step-like transitions of the inversion as a function of peak pulse strength are demonstrated to be observable in a QD exposed to a Gaussian pulse.

© 2005 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim

As a result of the strong coupling between incident electromagnetic field and atomic system, electron population oscillates between excited and ground states at the Rabi frequency [1, 2]. Rabi oscillations are well established in different physical systems, such as cold trapped ions [3], Bose–Einstein condensates [4], semiconductor quantum wells [5]. Among them, excitonic Rabi oscillations in quantum dots (QDs) are very promising: observed experimentally [6–8] they correspond to one-qubit rotation that is a step towards the QD application in quantum information processing [6]. Excitonic Rabi oscillations in a QD differ from those in an ordinary atom by following two factors: (i) oscillator strength in a QD is essentially larger [6]; (ii) Rabi oscillation picture strongly depends on the QD geometrical configuration. In recent investigations, essential role of the electron–hole dipole–dipole interaction (local fields) in the electromagnetic response of different structures has been predicted [9–15]. General equations describing interaction of quantum states of light with matter influenced by the local fields has been formulated, for the weak light-matter coupling regime, in [15] for bulk media and in [14] for confined QD excitons. The local field impact on the spontaneous emission of an excited two-level atom imbedded in the linear dielectric host has been considered in [15]. Paper [14] predicts local field induced fine structure of the QD absorption (emission) spectrum: instead of a single line with the frequency corresponding to the exciton transition, a doublet is appeared with one component shifted to the blue (red). In the limiting cases of classical light and single–photon states the doublet is reduced to a singlet shifted in the former case and unshifted in the latter one. One can expect that local field effects enhanced by the strong light-QD coupling will manifest themselves in a number of observable modifications of the conventional picture of the Rabi oscillations.

To describe the strong coupling between atom and electromagnetic field, the Jaynes–Cummings (JC) model is conventionally used [1, 2]. In our paper we incorporate into the JC model the electron–hole dipole–dipole interactions in QDs and present a corresponding microscopic theory of the excitonic Rabi oscillations. QD is modelled as a spatially confined two-level quantum oscillator. Coulomb interaction is assumed to be negligible. A 3D Cartesian basis $\boldsymbol{u}_\alpha$ ($\alpha = x, y, z$) is used, with the unit vector $\boldsymbol{u}_x$ parallel to the electron–hole pair dipole moment: $\mu = \mu \boldsymbol{u}_x$. We investigate an isolated arbitrary shaped QD exposed

[*] Corresponding author: e-mail: maksim@bsu.by

© 2005 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim

to nonharmonic electromagnetic field linearly polarized along μ: $\boldsymbol{E}(t) = E_{0x}\boldsymbol{u}_x = \mathrm{Re}\,[\mathcal{E}(t)\exp(-i\omega t)]\,\boldsymbol{u}_x$, where $\mathcal{E}(t)$ is the real-valued slow-varying amplitude of the electric field and ω is the field carrier frequency. For simplicity we further restrict ourselves to the case of classical incident field. Accordingly to [14], in the two-level approximation the QD exposed to the classical electromagnetic field is described by the Hamiltonian

$$H = (\varepsilon_e a_e^+ a_e + \varepsilon_g a_g^+ a_g) - V\hat{P}_x E_{0x} + \Delta H, \tag{1}$$

where $\varepsilon_{e,g}$ are the energy eigenvalues of the excited and ground states, respectively; $a_{g,e}^+$ and $a_{g,e}$ stand for the creation and annihilation operators of electron in the ground and excited states; V is the QD volume; $\hat{P}_x = V^{-1}(-\mu b^+ + \mu^* b)$ is the polarization operator; $b^+ = a_g a_e^+$ and $b = a_g^+ a_e$ are the creation and annihilation operators for the electron–hole pair. First term in (1) is the Hamiltonian of a free electron–hole pair, second term describes interaction of the pair with electromagnetic field, while ΔH is the correction to local fields. The latter one is given by [14]

$$\Delta H = 4\pi N_x P_x(-\mu b^+ + \mu^* b), \tag{2}$$

where N_x is the depolarization coefficient and $P_x = \langle \hat{P}_x \rangle$ is the macroscopic polarization of the QD.

Let us present wavefunction of the QD exposed to the electromagnetic field by

$$|\psi\rangle = \sum_{\sigma=+,-} A^\sigma |\sigma\rangle = A^+|+\rangle + A^-|-\rangle \tag{3}$$

where A^σ are the functions of time to be found and $|\sigma\rangle$ stands for the linear combination of ground and excited states: $|\pm\rangle = (|g\rangle \pm |e\rangle)/\sqrt{2}$. Since the incident field is assumed to be classical, wavefunction (3) does not contain photonic states; interaction with electromagnetic field is accumulated by functions A^σ. Equations for these functions are derived from the Shrödinger equation with Hamiltonian (1). For that aim, the Shrödinger equation with wavefunction (3) is projected onto the states $|\pm\rangle$. Then, utilizing the rotating-wave approximation one can obtain the equations of motion as follows:

$$\frac{\partial A^+}{\partial t} = i\frac{\Omega_R}{2}A^+ - i\frac{\Delta\omega}{2}[A^+|A^+|^2 - (A^+)^*(A^-)^2], \tag{4}$$

$$\frac{\partial A^-}{\partial t} = -i\frac{\Omega_R}{2}A^- - i\frac{\Delta\omega}{2}[A^-|A^-|^2 - (A^-)^*(A^+)^2]. \tag{5}$$

In these equations, the field carrier frequency ω is assumed to be equal to the exciton transition frequency ω_0; parameter $\Delta\omega = 4\pi N_x \mu^2/\hbar V$ is the local field-induced frequency shift [11].

Figure 1 demonstrates results of integration of Eqs. (4) and (5) for a ground-state QD exposed to the time-harmonic field ($E(t) = \mathrm{const}$). Exciton inversion dynamics (i.e., difference between populations of excited and ground states) for different field amplitudes defined by the parameter $\xi = \Omega_R/\Delta\omega$ has been numerically investigated. In Fig. 1, the inversion is plotted as a function of the dimensionless time $T = \Omega_R t/2\pi$. The calculations performed demonstrate strong dependence of the Rabi frequency on the depolarization parameter ξ. At $\xi \ll 1$ Rabi oscillations are practically disappear: $w(t) \sim \mathrm{const}$, that corresponds to the weak coupling between QD and electromagnetic field ($\Omega_R \to 0$). Small-amplitude oscillations of the inversion at $\xi = 0.2$, depicted in Fig. 1a, proves the foregoing statement. The situation is drastically changed in the vicinity of the value $\xi = \xi^{\mathrm{cr}} = 0.5$ which marks *bifurcation* in the oscillation dynamics (compare figures 1b and 1c). Thus, the value $\xi = \xi^{\mathrm{cr}} = 0.5$ separates two oscillatory regimes with drastically different characteristics. In the vicinity of the bifurcation, Rabi oscillations become *essentially anharmonic* (see Fig. 1c, d). This means that the Mollow triplet [2] (which characterizes Rabi oscillations in the frequency domain) is transformed into a more complicated spectrum with satellites corresponding to higher-order harmonics of the Rabi frequency. The anharmonism in the Rabi oscillations disappears with ξ further increase, see Fig. 1e. At $\xi > 1$, inversion behavior corresponds to the conventional picture of the Rabi oscillations $w(t) \cong \cos(2\pi T)$ [1, 2].

© 2005 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim

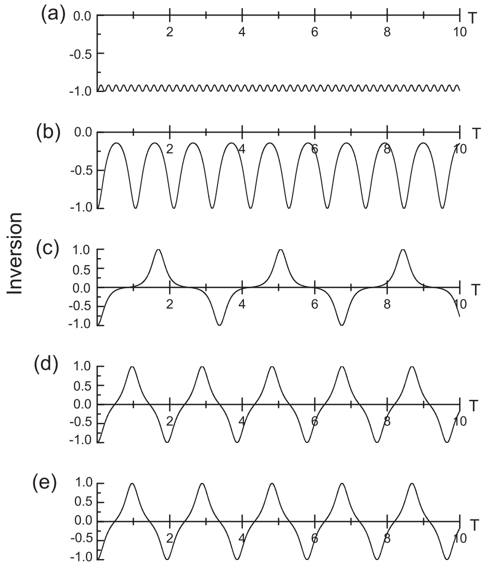

Fig. 1 Rabi oscillations of the inversion: $\xi = 0.2$ (a); $0.495\,(\mathrm{b})$; $0.5001\,(\mathrm{c})$; $0.51\,(\mathrm{d})$; $0.6\,(\mathrm{e})$.

Now we consider interaction of QD with the Gaussian pulse $E = E_0 \exp[-(t-t_0)^2/\tau^2]$ whose time duration τ is assumed to be much shorter relaxation times in the system. We let the Rabi frequency to be associated with the peak field strength of the Gaussian pulse: $E_0 = \mu\Omega_{R0}/\hbar$, so that $\xi = \Omega_{R0}/\Delta\omega$. Figure 2 shows the temporal dynamics of the inversion in the system. When $\xi \geq 1$, inversion demonstrates two different regimes of temporal evolution. In the first regime (Fig. 2a), the inversion final state w_f is the excited state $(t \to \infty, w \to 1)$. In the second regime (Fig. 2b), inversion returns back to the ground state $(t \to \infty, w \to -1)$. When $\xi \leq 1$ (that corresponds to small Rabi frequencies) only second regime is realized. It should be emphasized that the spontaneous emission which is not considered in our model would lead to decay of the final state of inversion in the first regime. However, even so the QD remains in the excited state for a long time. Indeed, for a spherical QD the spontaneous life-time of the excited state [18] can be associated, accordingly to Eq. (80) from Ref. [14], with the resonant frequency shift $\Delta\omega$ by:

$$T_{\mathrm{sp}} = \frac{\tau_{\mathrm{sp}}}{\tau} \simeq \frac{1}{4\tau\,\Delta\omega}\left(\frac{\lambda}{4\pi R_{\mathrm{QD}}\sqrt{\varepsilon_h}}\right)^3. \tag{6}$$

For the QD considered exposed to the Gaussian pulse with $\tau\,\Delta\omega = 15$ and wavelength $\lambda = 1.3$ μm, Eq. (6) gives for the above ratio the estimate $T_{\mathrm{sp}} \sim 1.3\times 10^2$. This result justifies neglect in our model the decay of the final excited state because of spontaneous emission.

Consider the final state of inversion as a function of the depolarization parameter ξ (See Fig. 3). At $\xi \geq 1 w_f(\xi)$ demonstrates step-like transitions between ground and excited states. The step width strongly

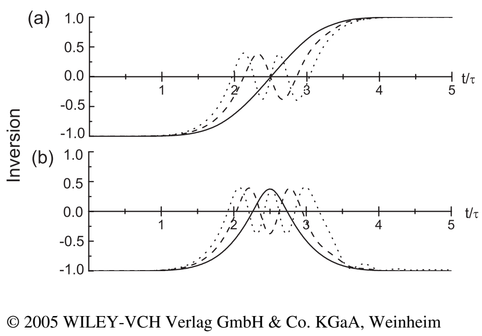

Fig. 2 Temporal dynamics of the inversion in a QD illuminated by a Gaussian pulse. $t_0 = 0.5\tau$, $\tau\,\Delta\omega = 15$. Two regimes of the dynamics are depicted: (a) $\xi = 1.0$ (solid line), $\xi = 1.26$ (dashed line), $\xi = 1.51$ (dotted line); and (b) $\xi = 1.13$ (solid line), $\xi = 1.39$ (dashed line), $\xi = 1.61$ (dotted line).

© 2005 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim

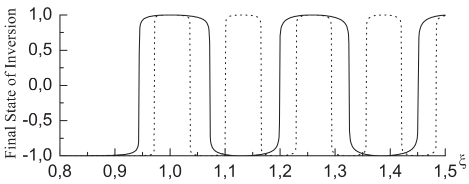

Fig. 3 The final state of inversion as a function of the peak field strength of the Gaussian pulse with $\tau\,\Delta\omega = 15$ (solid line) and $\tau\,\Delta\omega = 30$ (dotted line).

depends on the pulse duration. Similar effect for isotropic bulk media has been predicted in [16]. Such step-like transitions can be used in quantum information processing since the system has two stable states "0" and "1". Therefore, array of QDs controlled by applied field can potentially be used as a basis for logic operations.

Thus, local fields in QDs are predicted to entail following effects: (i) Bifurcation in the QD inversion; in the vicinity of that bifurcation Rabi oscillations are essentially anharmonic. (ii) The step-like transitions of the inversion as a function of the ultrashort optical pulse peak strength.

Acknowledgements The work was partially supported by the NATO Science for Peace Program under project Sfp-972614 and NATO Linkage Grant under project PST.CLG.980375.

References

[1] W. P. Schleich, Quantum Optics in Phase Space (Wiley, Berlin, 2001).
[2] M. O. Scully and M. S. Zubairy, Quantum Optics (Cambridge, University Press, 1997).
[3] A. A. Budini, R. L. de Matos Filho, and N. Zagury, Phys. Rev. A **67**, 033815 (2003).
[4] M. Kasevish, Science **298**, 1363 (2002).
[5] A. Schulzgen et al., Phys. Rev. Lett. **82**, 2346 (1999).
[6] T. H. Stievater et al., Phys. Rev. Lett. **87**, 133603 (2001).
[7] H. Kamada et al., Phys. Rev. Lett. **87**, 246401 (2001).
[8] J. Forstner et al., Phys. Rev. Lett. **91**, 127401 (2003).
[9] S. Schmitt-Rink, D. A. B. Miller, and D. S. Chemla, Phys. Rev. B **35**, 8113 (1987).
[10] B. Hanewinkel, A. Knorr, P. Thomas, and S. W. Koch, Phys. Rev. B **55**, 13715 (1997).
[11] S. A. Maksimenko et al., Mat. Sci. Engineer. B **82**, 215 (2001).
[12] G. Ya. Slepyan et al., Phys. Rev. B **59**, 12275 (1999).
[13] S. A. Maksimenko et al., J. Electronic Materials, **29**, 494 (2000).
[14] G. Ya. Slepyan, S. A. Maksimenko, A. Hoffmann, and D. Bimberg, Phys. Rev. A **66**, 063804 (2002).
[15] M. E. Crenshaw and C. M. Bowden, Phys. Rev. Lett. **85**, 1851 (2000); Phys. Rev. A **63**, 013801 (2001).
[16] M. E. Crenshaw, M. Scalora, and C. M. Bowden, Phys. Rev. Lett. **68**, 911 (1992).
[17] Y. Fu, M. Willander, and E. L. Ivchenko, Supperlattices and Microstructures **27**, 255 (2000).
[18] L. D. Landau and E. M. Lifshitz, Electrodynamics of continuous media (Pergamon Press, Oxford, 1960).
[19] G. Khitrova et al., Rev. Mod. Phys. **71**, 1591 (1999).

© 2005 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim