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Light confinement in a quantum dot

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Abstract. The phenomenon of light confinement in an isolated quantum dot, originating from the electromagnetic wave diffraction at the dot boundary, is discussed. It has been shown that at a certain condition the quantum dot behaves as a microcavity whose eigenmodes manifest themselves as additional, geometrical, resonances in the quantum dot electromagnetic response. The effect of induced magnetization of the quantum dot is predicted and illustrated by the example of magnetic resonances in spherical quantum dots. The radiative lifetime for a spherical quantum dot has been evaluated and a correlation has been discussed between radiative lifetimes in quantum dots and quantum wells.

1. Introduction

Rapid progress in the synthesis of a variety of different kinds of nanostructure with fascinating electronic and optical properties irreducible to properties of bulk media symbolizes a fundamental breakthrough in solid state physics. The idea of using structures with size quantization of charge carriers in one or more directions as active media of a double heterostructure laser has been proposed in [1]. Particularly significant improvements were expected for lasers based on 3D confined narrow-gap insertions in a host semiconductor, quantum dots (QDs) [2, 3]. However, practical development of a QD laser was limited by the non-availability of fabrication techniques for dense arrays of QDs uniform in shape and size free from defects and dislocations. Such a technique suffering from high concentration of dislocated clusters [5], has been elaborated on the basis of self-organized formation of nanoscale islands [4]. More recently this technique has been developed to a level when laser generation via QD states up to room temperature became possible [6]. Kirstaedter *et al* [7] demonstrated very low threshold current density and its practically complete temperature insensitivity in an InGaAs/GaAs QD laser up to 180 K. Ultrahigh material and differential gain, exceeding by orders of magnitude those in QW lasers, has been experimentally confirmed [8].

The key peculiarity of QDs is related to the 3D confinement of the electron motion with discrete energy levels determined by QD size and shape. To elucidate the role of the confinement and to utilize it in QD-based devices, a large body of research work, both theoretical and experimental (e.g., [8] and references therein), has been undertaken in recent years. However, there exists a class of effects governed by the QD shape, which have not received much attention so far. These effects are related to the electromagnetic wave diffraction by QDs, which drastically influences the electromagnetic response properties of QDs. Indeed, the resonant nature of the exciton provides a dramatic resonant discontinuity of the dielectric function at the QD boundary and, consequently, electromagnetic field diffraction by the QD. Diffraction modifies the electromagnetic field structure both inside and outside the QD and makes this field inhomogeneous on the QD extension scale (5–10 nm). Note that this extension is much less than the optical wavelength in the host medium. Thus, a coupled exciton–photon state (CEPS) is formed in a QD exposed to the external electromagnetic field. Different from polaritons in bulk media, this state is localized in the QD, exhibits discrete spectrum and cannot be characterized by a certain momentum. Thus, by analogy with the charge carrier confinement, this phenomenon can be referred to as *light confinement*.

To our knowledge, the role of light confinement in an individual QD was first considered by Schmitt-Rink *et al* [9]. The manifestation of this phenomenon in geometrically complex mesoscopic and subwavelength systems was discussed by Martin *et al* [10] in relation to the scanning near-field optical microscopy of such systems. The role of the diffraction-induced depolarization fields for the matching of electromagnetic boundary conditions has been pointed out in this reference. Formation of inhomogeneous near fields of QD responsible for interaction of the QD with the tip in scanning near-field microscopes has been considered by Hanewinkel *et al* [11]. Due to the depolarization field, the CEPS resonance in the QD proves to be shifted with respect to the exciton resonant frequency [11, 12]. In [11] it was noted that the optical absorption and gain of a single QD could be distinguished owing to the shift, blue in the former case and red in the latter one. Electromagnetic field diffraction by anisotropically shaped QDs provides polarization splitting of the gain band in QD arrays. This effect was predicted and experimentally verified by Slepyan *et al* [12]. Note also that the depolarization field effect has been proposed by Gammon *et al* [13] as a hypothesis explaining the experimentally observed polarization-dependent splitting of the PL spectrum of a single anisotropically shaped QD.

Formation of a CEPS in a QD and inhomogeneity of the electromagnetic field in its vicinity is of importance for a theoretical estimate of the radiative lifetime in QDs. As applied to QWs, this problem was considered in [14–17]. Field inhomogeneity at boundaries in planar structures, QWs, is described either by the Fresnel formula [16] or by their equivalent boundary conditions of a special kind [14]. Such an approach is inapplicable for QDs: the boundary-value problem for the Maxwell equations for QDs is much more complicated. This problem admits analytical treatment only in a number of particular cases, for highly symmetrical QDs (sphere, spheroid). An estimate of the radiative lifetime in disc-like QDs has been performed by Sugurava [18] on the basis of microscopic theory. However, Maxwell equations have not been attracted for evaluation of real structure of the electromagnetic field. Instead, intuitive assumptions concerning the scalar potential behaviour were used.

Thus, the light confinement plays an important role in formation of the QD electromagnetic response and must be taken into account for its correct prediction. In this paper we discuss some general consequences of this phenomenon in an isolated QD (section 2). In section 3 we extend the analysis beyond the dipole approximation of the diffraction theory, which was used in previous treatments [11, 12]. This approximation assumes the QD linear extension much less as compared to the wavelength in both host medium and QD itself, and corresponds to the ground state of the confined CEPS. We show that in the situation where such an assumption is invalid, upper CEPS levels are excited resulting in the *induced magnetism* of QDs. An estimate of the radiative lifetime in spherical QDs and comparison with QWs is presented in section 4. Concluding remarks are presented in section 5.

2. Depolarization of quantum dots

To elucidate basic peculiarities of the QD electromagnetic response caused by the diffraction, we make use the simplest phenomenological model of a QD considering exciton as a Lorentzian contribution to the dielectric constant ε_h of the QD material (see, e.g., [19]) and restricting ourselves to the single-resonance approximation:

$$\varepsilon(\omega) = \varepsilon_h + \frac{g_0}{\omega - \omega_0 + \mathrm{i}/\tau}. \qquad (1)$$

A comprehensive analysis of the applicability of the this model to excitons in semiconductor heterostructures has been presented by Pau *et al* [20]. In equation (1), ω_0 is the exciton resonant frequency; τ is the exciton lifetime. The parameter g_0 is related to the oscillator strength per dot, f_0, by $g_0 = f_0\lambda e^2/m_e c$, where e and m_e are the electron charge and mass, while λ and c are the vacuum wavelength and the light velocity. Furthermore, ε_h is assumed to be real, frequency independent and equal to the dielectric constant of the host material. The case $g_0 > 0$ corresponds to QD with inverted population of levels, thus characterizing optical gain. Note that parameter g_0 also depends on the QD radius [8] and this dependence can be extracted from the rigorous quantum-mechanical simulation. Here, we restrict ourselves to a following simple estimate of g_0 using experimental data. The parameter g_0 is related to the material gain per dot $\tilde{g}$ by $\tilde{g} \approx kg_0/2\Gamma_{inh}\sqrt{\varepsilon_h}$, where Γ_{inh} is the inhomogeneous broadening width in the measured QD sample. The value of $\tilde{g}$ is extracted from experimental measurements of the QD ensemble gain. Letting $\tilde{g} \equiv 10^5$ cm^{-1} [8, 21] and $\Gamma_{inh} \cong 10^{13}$ s^{-1} [8], for $\lambda = 1300$ nm one can obtain $|g_0| \cong 10^{14}$ s^{-1} that correlates to the theoretical prediction [3].

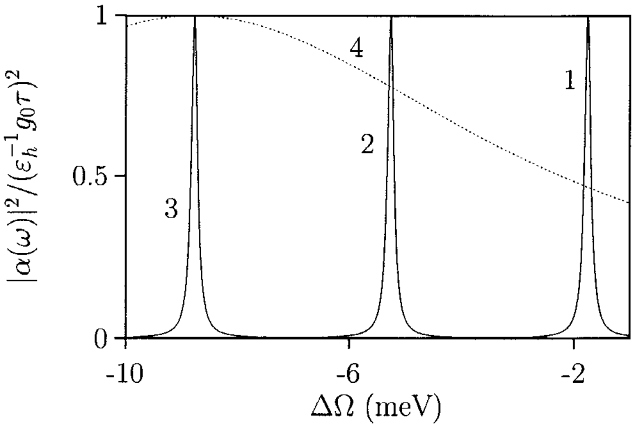

Figure 1. Shift of the normalized polarizability $|\alpha|^2/(\varepsilon_h^{-1}g_0\tau)^2$ as a function of $\Delta\Omega = \omega - \omega_0$ with the parameter g_0 for the main CEPS resonance of a spherical QD. $\tau = 10^{-11}$ s, $g_0 = 1\times10^{14}$ (1), 2.5×10^{14} (2), 5×10^{14} (3) s^{-1}. Curve 4, $\tau = 10^{-13}$ s, $g_0 = 5\times10^{14}$ s^{-1}.

The electromagnetic response of a medium is determined by its polarization under the effect of an external electromagnetic field. In bulk materials the polarization is given by $\boldsymbol{P} = (\varepsilon(\omega)/\varepsilon_h - 1)\boldsymbol{E}/4\pi$. Since the QD linear extension is a small quantity as compared to the wavelength, the equation $\boldsymbol{P} = \alpha(\omega)\boldsymbol{E}/4\pi$ [22] must be applied instead, introducing the polarizability $\alpha(\omega)$ as the response characteristics of an electrically small object exposed to the electromagnetic field. Both the shape of the scattering object and its material properties given by equation (1) determine the polarizability. As a result, $\alpha(\omega)$ is, generally, a tensor in spite of the fact that $\varepsilon(\omega)$ and ε_h are scalars. Thus, equation (1), which assumes an infinite, isotropic, homogeneous medium, can serve only as a rough approximation to the case under consideration: the polarizability of a single QD is distinct from that given by equation (1) because of the effect of electromagnetic field diffraction. In the dipole approximation, applicable when the QD extension R is much less as compared to the wavelength both outside and *inside* the QD, the condition $|\kappa(\omega)| \ll 1$

holds true, where

$$\kappa(\omega) = kR\sqrt{\varepsilon(\omega)} \tag{2}$$

and $k = \omega/c$. This condition allows one to account for the diffraction process by means of the depolarization field [22], so that the electric field vector inside the QD is represented by $\boldsymbol{E}_{QD} = \boldsymbol{E} - 4\pi N\boldsymbol{P}$, with N as the depolarization tensor completely determined by the QD shape. In that case, the polarizability components $\alpha_\sigma(\omega)$ in the main axes of a certain shaped QD are given by [22]

$$\alpha_\sigma(\omega) = \left[\frac{\varepsilon_h}{\varepsilon(\omega) - \varepsilon_h} + N_\sigma\right]^{-1} \tag{3}$$

where N_σ are the corresponding components of the depolarization tensor. As follows from equation (3), the depolarization field shifts the QD resonant frequency from the exciton resonance to the CEPS ones, given by [11, 12]

$$\omega_N^\sigma = \omega_0 - \frac{g_0}{\varepsilon_h}N_\sigma. \tag{4}$$

Note that a similar shift has been predicted for surface polaritons in quantum wells [17]. The depolarization shift is proportional to the oscillator strength and has opposite signs for absorbing and amplifying QDs. The red shift in the inverted QD can easily be understood by referring to the fact that the irradiation of a photon from the QD requires the photon–exciton bond to disrupt. The opposite situation is characteristic for absorbing QDs. For spherical QDs considered in [11] (and also for cubic ones), $N = 1/3$ and $\alpha(\omega)$ is a scalar function; in this case, the CEPS resonance is doubly degenerate. A distinction between N_σ in anisotropically shaped QDs provides the polarization-dependent shift described in [12]. Figure 1 illustrates the frequency shift of the polarizability $\alpha(\omega)$ with the parameter g_0 for a single spherical QD. The range of the parameter g_0 variation has been chosen on the basis of the estimate given above. The quantity $\tau = 10^{-11}$ s represents typical dephasing time in an isolated QD [8]. Inhomogeneous broadening in QD arrays reduces the dephasing time to the level $\sim 1/\Gamma_{inh}$ applied for the curve 4 calculation. As can be seen, the shift exceeds the linewidth even in the case when this linewidth corresponds to the inhomogeneous broadening.

The above reasoning leads to some important conclusions concerning the role of light confinement in QDs:

(i) The CEPS frequencies ω_N^σ (or, dependent on the QD symmetry, the degenerate frequency), but not the exciton frequency ω_0, are experimentally observed in QD optical spectra and conventionally identified as the exciton frequency.

(ii) The depolarization shift is completely determined by the shape of QDs and does not depend on the properties of QD arrays in measured samples (volume fraction, lattice configuration etc).

(iii) The difference between exciton and CEPS frequencies strongly influences the polarization effects in asymmetric QDs and must be taken into account for their interpretation [12], including expected experiments on polarization splitting in microcavities with QDs.

(iv) The difference between the absorption and the gain CEPS resonances [11], predicted to be observable even for inhomogeneously broadened lines (curve 4 in figure 1), is of importance for interpretation of photoluminescence experiments.

(v) Electromagnetic confinement is not taken into account in quantum-mechanical modelling of the exciton levels in QDs. A rigorous approach to the problem must include self-consistent solution of the Liouville equation for the density matrix in the QD and Maxwell equations for the electromagnetic field.

Depolarization defines the ground CEPS level in the QD. The manifestation of upper levels is discussed in the next section.

3. Radiative lifetime of spherical QDs

Let an isolated spherical QD of radius R be exposed to the external electromagnetic field $\{\boldsymbol{E}_0(\boldsymbol{r}), \boldsymbol{H}_0(\boldsymbol{r})\}\exp(-\mathrm{i}\omega t)$. The problem of wave diffraction by a sphere was exactly solved early last century [23, 24] by using the variable separation in the spherical basis. In view of the condition $kR\sqrt{\varepsilon_h} \ll 1$, which is valid for any realistic QDs, this solution is essentially simplified [25] and presents the field outside the sphere by:

$$\begin{Bmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{Bmatrix} = (\nabla\nabla + \varepsilon_h k^2)\begin{Bmatrix} \boldsymbol{\Pi}^e \\ \boldsymbol{\Pi}^m \end{Bmatrix} + \mathrm{i}k\nabla \times \begin{Bmatrix} \boldsymbol{\Pi}^m \\ -\sqrt{\varepsilon_h}\boldsymbol{\Pi}^e \end{Bmatrix} \tag{5}$$

where Hertz potentials are given by:

$$\begin{Bmatrix} \boldsymbol{\Pi}^e(r) \\ \boldsymbol{\Pi}^m(r) \end{Bmatrix} = \frac{R^3}{3r}\begin{Bmatrix} \alpha^e\boldsymbol{E}_0(0) \\ \alpha^m\boldsymbol{H}_0(0) \end{Bmatrix}\exp(\mathrm{i}k\sqrt{\varepsilon_h}r) \tag{6}$$

and the electric and magnetic polarizabilities of the sphere $\alpha^{e,m}(\omega)$ are as follows:

$$\alpha^e(\omega) = 3\frac{\varepsilon(\omega)F(\kappa) - \varepsilon_h}{[\varepsilon(\omega)F(\kappa) + 2\varepsilon_h](1 - \mathrm{i}kR\sqrt{\varepsilon_h}) + \mathrm{i}(kR)^2\varepsilon_h^2F(\kappa)}$$

$$\alpha^m(\omega) = 3\frac{F(\kappa) - 1}{[F(\kappa) + 2](1 - \mathrm{i}kR) + \mathrm{i}(kR)^2F(\kappa)}. \tag{7}$$

The function

$$F(\kappa) = 2\frac{\sin\kappa - \kappa\cos\kappa}{(\kappa^2 - 1)\sin\kappa + \kappa\cos\kappa} \tag{8}$$

with $\kappa \equiv \kappa(\omega)$ defined by equation (2), is responsible for the diffraction effect. It can easily be found that the above expression for $\alpha^e(\omega)$ is reduced to equation (3) with $N_\sigma = 1/3$ by substituting $F(\kappa) = 1$ and neglecting the small imaginary terms in the denominator. It should be pointed out that these imaginary terms are of importance for evaluation of the radiative lifetime. Note that the solution (5)–(8) does not contain parameters characterizing the spatial configuration of incident electromagnetic field (e.g., angle of incidence for plane waves). This allows one to extend the analysis to an arbitrary configured incident field only assuming the field is slowly varied in the QD volume. The last restriction is valid in all realistic situations.

Since we have restricted ourselves to the case of an isolated QD whose electromagnetic response cannot be described by the dielectric function, the conventional gain concept, $g(\omega) = -(\omega/c)\Im[\varepsilon(\omega)]$, cannot be applied for characterization of the QD. The flux of irradiated energy is adequate in that case. The energy flux through a spherical surface surrounding the QD is given by

$$\Sigma(\omega) = -\frac{c}{8\pi}\Re\oint_S [\boldsymbol{E}\times\boldsymbol{H}^*]\boldsymbol{e}\,\mathrm{d}S$$
$$= -\frac{\omega\varepsilon_h^2}{6g_0\tau}R^3\{|\alpha^e(\omega)|^2|\boldsymbol{E}_0(0)|^2 + |\alpha^m(\omega)|^2|\boldsymbol{H}_0(0)|^2\}. \qquad (9)$$

Note that $\Sigma(\omega) < 0$ for $g_0 < 0$. This corresponds to light absorption by the QD. In the opposite case, when $g_0 > 0$, we deal with light emission by an inverted QD: $\Sigma(\omega) > 0$. It follows from equation (9) that the structure of QD gain and absorption spectra is completely determined by the frequency dependences of the quantities $|\alpha^{e,m}(\omega)|^2$.

In QWs, the problem of the radiative lifetime evaluation is solved by finding frequency poles of the reflection and transmission coefficients for TE- and TM-polarized plane waves at given angles of incidence [14–17]. The real parts of these poles determine the resonant frequencies while the imaginary parts give the homogeneous linewidths, which are sums of the dephasing broadening and the radiative broadening. An analogous approach can be applied to QDs. The only difference is that we must evaluate the poles of the electric and magnetic polarizabilities given by equation (7). In the case of the main CEPS resonance, where $F(\kappa)\to 1$, simple manipulations lead to

$$\tau_{rad}^{QD} \cong -\frac{9}{4\pi^2 g_0}\left(\frac{\lambda}{R}\right)^2. \qquad (10)$$

Here $\tau_{rad}^{QD} > 0$ for absorbing QDs and $\tau_{rad}^{QD} < 0$ for amplifying ones. Substitution of $g_0 \cong -10^{14}$ s^{-1} and $R = 2.5$ nm into equation (10) gives $\tau_{rad}^{QD} \cong 6.2\times 10^{-10}$ s for $\lambda = 1300$ nm. This magnitude is in the range of experimental values of the radiative lifetime measured in [26] for pyramidal QDs. Furthermore, our estimate correlates with that obtained by Asryan and Suris [27] for cubic QDs from quantum-mechanical consideration without accounting for the light confinement. This allows us to suppose that the light confinement mainly manifests itself in the depolarization shift of the exciton resonant frequency and gives a certain contribution to the radiative lifetime. For a more correct prediction, we must discard the idealistic spherical model of the QD shape and take into account its real geometry as well as the radial dependence of g_0. In any case, equation (10) shows *additional* radial dependence of the radiative lifetime, explaining its decrease as the QD linear extension grows.

For the main CEPS resonance, the spectrum of the energy flux is expressed by

$$\frac{\Sigma_\sigma^{QD}(\omega)}{\Sigma_0^{QD}} = \frac{6}{kR\sqrt{\varepsilon_h}}\frac{\Gamma_{rad}^{QD}\gamma}{(\omega-\omega_N^\sigma)^2 + (\Gamma_{rad}^{QD}+\gamma)^2} \qquad (11)$$

where $\Gamma_{rad}^{QD} = (\tau_{rad}^{QD})^{-1}$ and $\gamma = \tau^{-1}$. The normalization coefficient $\Sigma_0^{QD} = c\sqrt{\varepsilon_h}R^2|\boldsymbol{E}_0(0)|^2/4$ is the incident energy flux passing through the hemisphere of radius R.

Let us correlate now equations (10) and (11) with analogous estimates for QWs. The radiative time in QWs is given (see, e.g., [20]) by

$$\tau_{rad}^{QW} \cong -\frac{\sqrt{\varepsilon_h}}{\pi g_0}\frac{\lambda}{L} \qquad (12)$$

where L is the QW thickness, whereas the corresponding expression for the energy flux is as follows:

$$\frac{\Sigma_\sigma^{QW}(\omega)}{\Sigma_0^{QW}} = \frac{2\Gamma_{rad}^{QW}\gamma}{(\omega-\omega_0)^2 + (\Gamma_{rad}^{QW}+\gamma)^2}. \qquad (13)$$

Here $\Gamma_{rad}^{QW} = (\tau_{rad}^{QW})^{-1}$; $\Sigma^{QW}(\omega)$ and Σ_0^{QW} are the energy fluxes of scattered and incident waves through the unit surface of the QW. Since equation (10) contains the ratio $\lambda/R \gg 1$ in the square while equation (12) is linear with respect to it, the relation $\tau_{rad}^{QD} \gg \tau_{rad}^{QW}$ holds true at equal g_0, ε_h and $L = 2R$. Furthermore, even at equal radiative times the normalized energy flux for QDs (11) exceeds significantly the normalized flux for QWs owing to the multiplier $3/kR\sqrt{\varepsilon_h} \gg 1$, which reflects the role of light confinement in QDs. Thus, the radiative (absorbing) efficiency of QDs turns out to be much higher as compared to QWs, providing fascinating perspectives of QD lasers.

4. Geometrical resonances in spherical QD

From the above stated condition, $|\kappa(\omega)| \ll 1$, the role of diffraction by a QD is irreducible to the effect of depolarization. At $|\kappa| > 1$ the wavelength inside the QD becomes comparable with its linear extension, and, as follows from equations (5)–(8), the scattered wave field is generated by irradiation of both electric and *magnetic* dipoles indicating thus *induced magnetism* of QDs. The physical mechanism of magnetization of dielectrics with linear extension compared with the internal wavelength is related to the excitation of internal TE_{1q1} cavity modes ($q = \pm 1, \pm 2, \ldots$ are the polar indices of the modes) in a scattering object, which thus behaves as a microcavity. Such modes give rise to a curl electric current in its turn inducing nonzero magnetic moment of the object [22]. The given effect is known in macroscopic electrodynamics; it is observed in macroscopic dielectric composite materials [28]. A peculiarity of the magnetism in QDs is its pronounced resonant nature. The eigenmodes indicated are called geometrical resonances. The term 'geometrical' [25] is related to the fact that the resonances occur exceptionally owing to a certain geometrical configuration of the QD.

The resonant conditions for electric and magnetic geometrical resonances,

$$\varepsilon(\omega)F(\kappa) + 2\varepsilon_h = 0 \qquad \text{and} \qquad F(\kappa) + 2 = 0$$

follow from equation (7) after neglecting imaginary corrections. These conditions are completely determined by the properties of the function $F(\kappa)$. Figure 2 demonstrates the behaviour of the function $F(\kappa)$ versus $\Delta\omega = \omega - \omega_0$ at different values of the input parameters g_0 and τ. There exists a set of resonances in the vicinity of the exciton frequency, whereas $\Re[F(\kappa)] \to 1$ and $\Im[F(\kappa)] \to 0$ at

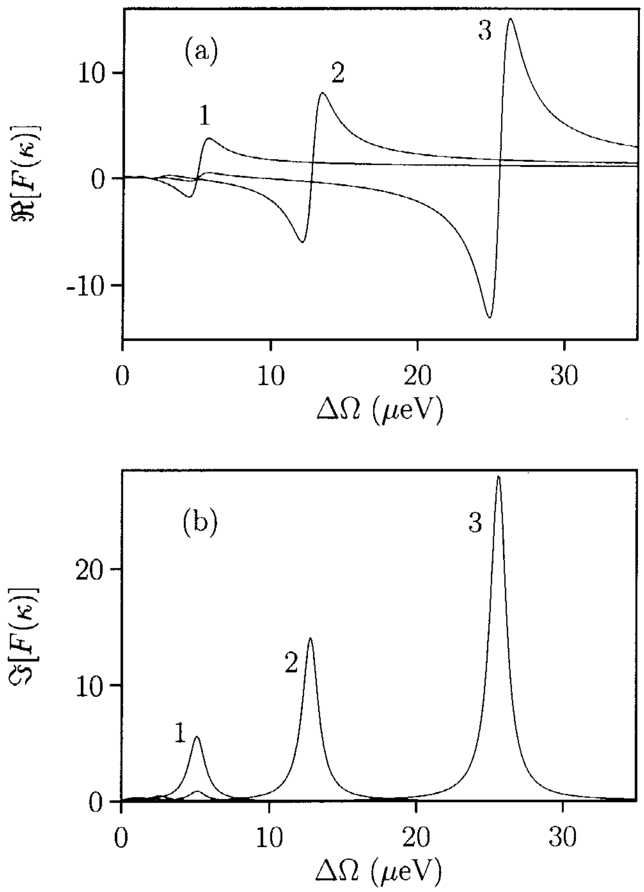

Figure 2. Frequency dependence of the diffraction function $F(\kappa)$ in the vicinity of exciton resonance. $\tau = 1 \times 10^{-9}$ s, $g_0 = 1 \times 10^{14}$ (1), 2.5×10^{14} (2), 5×10^{14} (3) s^{-1}, $R = 5$ nm.

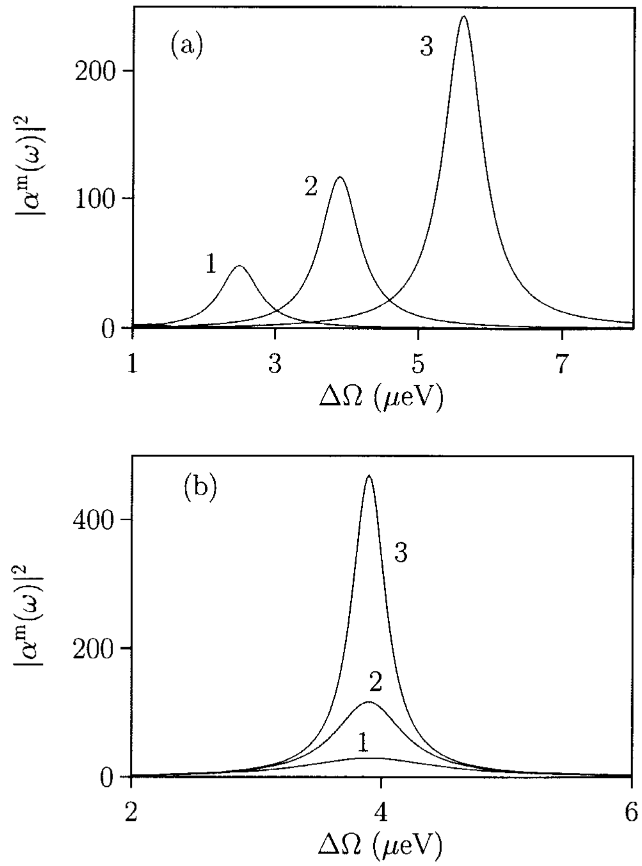

Figure 3. Magnetic resonances in an isolated QD. (a) $\tau = 2 \times 10^{-9}$ s, $g_0 = 1 \times 10^{14}$ s^{-1}, $R = 4$ (1), 5 (2), 6 nm (3). (b) $\tau = 1 \times 10^{-9}$ (1), 2×10^{-9} (2), 4×10^{-9} (3) s, $g_0 = 1 \times 10^{14}$ s^{-1}, $R = 5$ nm.

$|\Delta\omega| \to \infty$. The last tendency is also inherent in this function as $g_0\tau$ grows smaller reducing the problem to that considered in [9], [11] and [12]. Thus, one can conclude that the geometrical resonances can manifest themselves in the vicinity of the exciton frequency and certainly disappear far away this frequency region. To evaluate the radiative lifetime of the geometrical resonances we make use the procedure applied earlier for evaluation of τ_{rad}^{QD} (10). For magnetic geometrical resonance it gives

$$\tau_{rad}^{M} \cong -\frac{1}{32 g_0}\left(\frac{\lambda}{R}\right)^4. \tag{14}$$

For input values $g_0 \cong -10^{14}$ s^{-1}, $R = 2.5$ nm and $\lambda = 1300$ nm we obtain $\tau_{rad}^{M} \cong 2.3 \times 10^{-5}$ s while $\tau_{rad}^{M} \cong 1.4 \times 10^{-6}$ s for a QD with $R = 5$ nm. Thus, the magnetic resonance exhibits much longer radiative lifetime as compared to the main CEPS resonance. Furthermore, this lifetime is much longer than the intrinsic dephasing time, which therefore is crucial for the possibility to observe the magnetic resonance. Assuming the dephasing time at low temperature to be of the order of 10^{-8}–10^{-9} s [18, 19], we can put this estimate in equation (1) and make use of equation (7) for calculation of $|\alpha^m(\omega)|^2$. Note that $g_0\tau \cong 10^6$–10^7 for the chosen g_0 magnitude. This signals a substantial difference between ε_h and $\varepsilon(\omega)$ in the vicinity of the magnetic resonance: $|\kappa(\omega_0)|$ lies in the range between 4 and 40 for spherical QDs with the effective diameter 5 nm.

Figure 3 presents $|\alpha^m(\omega)|^2$ for an inverted QD at different values of R (a) and τ (b). Magnetic resonance demonstrates the blue shift with respect to ω_0. The magnitude of the shift is determined by g_0 and is independent of τ. Since $\tau \ll \tau_{rad}^{M}$, the peak width is completely determined by the dephasing time. The procedure, which led us to the above estimate of g_0 from the experimentally determined value of the gain per dot $\tilde{g}$, being carried out in the reverse order, leads to $\tilde{g} \sim 10^2$ cm^{-1} for magnetic gain per dot. A rigorous averaging procedure must be applied for a more correct estimate.

Concerning electrical geometrical resonances we have to conclude that they are not of interest because they can not be excited separately from the main CEPS resonance depicted in figure 1. This is because both types of electric resonances are excited by the electric component of the external field. Since the intensity of electrical geometrical resonance is a small portion of the main CEPS resonance intensity, its contribution results in small-amplitude beatings on the main line slope. Thus, higher electrical eigenmodes practically do not influence the main (depolarization) resonance. Such behaviour can easily be understood by referring to equation (8), which shows the property $F(\kappa) = 1$ in the vicinity of ω_N.

The occurrence of the magnetic geometrical resonance in isolated QDs must lead to magnetization of a QD array in the vicinity of the exciton frequency, essentially shifted to the blue with respect to the CEPS resonance observable in experiments. A rough estimate of the array permeability can be obtained from the equation $\mu = 1 + f_V\alpha^m$, where f_V is the volume fraction of QDs in the array. Using realistic parameters, one can find $\mu - 1 \sim 0.05$–0.1. This is available for observation. For a more correct estimate, effects of inhomogeneous broadening and Maxwell Garnet homogenization formalism must be involved in the analysis.

Thus, we can conclude that the electromagnetic wave diffraction by QDs may result in manifestation by QD arrays of magnetic properties although both the QD and surrounding materials are dielectric. Note that the placement of a QD in a microcavity in an antinode of a magnetic field creates a possibility to make the effect evident without excitation of the main resonance.

5. Conclusion

In this paper we have investigated the role of light confinement in electromagnetic response properties of isolated QDs. We calculated significant diffraction-induced shift of the main QD exciton line. The occurrence of magnetic geometrical resonances caused by the excitation of eigenmodes in QDs, which thus behave as microcavities, is predicted. Having much smaller intensity as compared to the main exciton peak, these resonances can be evident owing to their shifts with respect to the main CEPS peak and can be excited by placing the QD in a microcavity in the magnetic field antinode, where the main peak is suppressed. Measurement of the frequency shift between the main CEPS and magnetic resonances can be used for direct determination of the oscillator strength in QDs. Evaluation of the radiative lifetime for a spherical QD and its correlation to the QW radiative lifetime shows the origin of the fascinating light-amplifying properties of QDs as compared to QWs and creates a basis for solving a large number of electrodynamic problems of QDs and QD ensembles.

In our paper we restricted ourselves to the spherical model of QDs. Different QD configurations such as discs or pyramids can be investigated using direct computation on the basis of the well developed method of classical electrodynamics [29].

Acknowledgments

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